8 Using graphs and diagrams

Numeracy extends beyond simply using mathematical techniques, and includes the interpretation of quantitative data in a real world context, problem solving, and the communication of results in a form useful for decision making.

It is now easy to collect huge quantities of data, particularly with the widespread use of computers in all areas of society. For example, many different readings may be made at frequent time intervals at hundreds of weather stations, or sales data may be collected for thousands of different products in different shops. Graphs and diagrams are often necessary when interpreting large quantities of data, so that patterns or trends can be identified.

Graphs and diagrams in critical thinking

Critical thinking is the ability to analyse arguments, perhaps presented by people with widely differing views. Quantitative data can provide an important resource when developing a convincing logical argument. Evidence may already exist in the form of graphs or diagrams, or data in a numerical format may first need to be plotted as graphs or diagrams to help in its interpretation.

When researching an issue, it is important to obtain data which is reliable. Various guiding principles can help in this process. We might be more confident in using data which is:

- **corroborated** by more than one source, for example when *most* scientists or economists believe that a particular result is correct
- obtained from a source with a good **reputation** for accuracy, such as a university or the BBC.
- collected by persons who have had first-hand **access** to events or original documents, rather than simply reporting information provided by other people.
- obtained from a person or organisation with no **vested interest**. We might be cautious, for example, of health data provided by a fast food company or soft drinks manufacturer.
- obtained from an **expert** in the field. For example, we might give greater credibility to data about the cause of an air crash provided by an aviation specialist.
- obtained from a person or organisation expected to be **neutral** with regard to the issue. We might be cautious about accepting data on religious matters from a devout member of a particular religious faith, or political data from a politician in a particular political party.

To examine how these principles might be applied, and how data can be presented as informative graphs and diagrams, we will consider some important questions relating to the topic of **climate change**. For example, we might investigate the questions: Is climate change occurring? If so, is this the result of human activities? What consequences might climate change have for people living in different parts of the world?

A starting point for the investigation might be to examine changes in average global temperatures over a long period, up to the present day. Figure 181 is a graph of temperatures produced by the NASA Goddard Institute for Space Studies. We might expect that this institute has a good reputation for accurate work, and that it employs scientists with a high level of expertise. The data displayed is therefore likely to be very reliable.

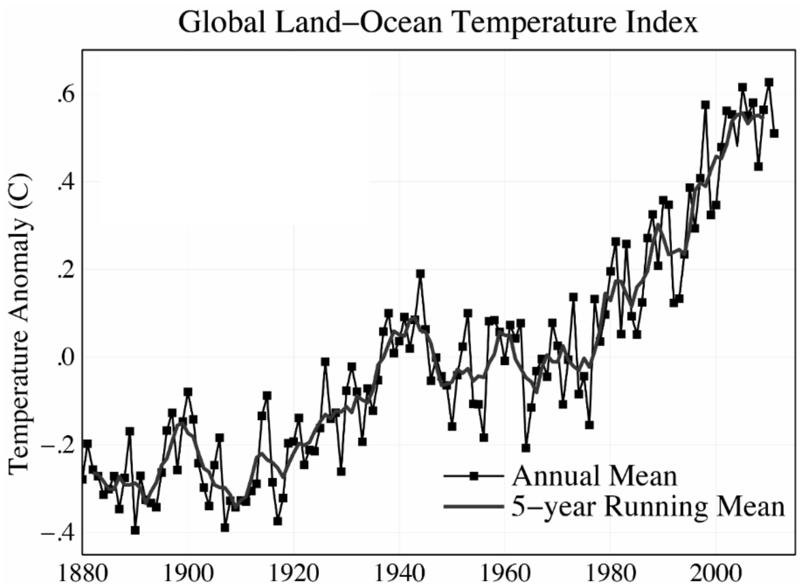


Figure 181: Global average land-ocean temperatures for the period 1880 - 2010

A rise in temperature seems to have definitely occurred over the past 100 years, but the absolute temerature rise of less than 1°C seems small. This may not yet be significant, but there may be cause for concern if temperature continues to rise at the same rate or at an increased rate.

We might now investigate changing patterns of rainfall. There is evidence of some extreme rainfall occurring in Britain in recent years. Figure 182, for example, shows a comparison between actual rainfall during the month of December 2015 and the average December rainfall over the previous 30 years. This map is produced by the Meteorological Office using data collected at weather stations across Britain. We would expect this map to be accurate, as the organisation has a good reputation and employs expert staff.

The December 2015 map shows that many areas of Britain experienced more than twice the expected rainfall for that month, leading to extensive and serious flooding. We will need to investigate whether this exceptional rainfall was just a chance event, or the result of a long term trend of increasing rainfall over western and northern Britain.

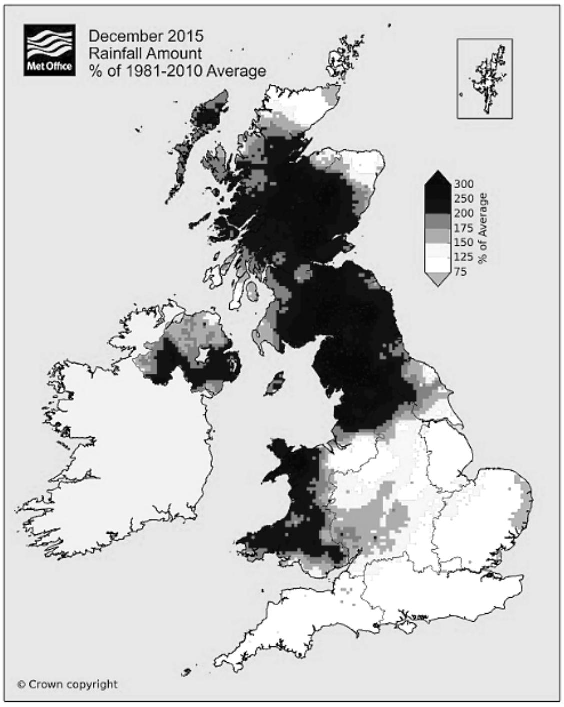


Figure 182:

Rainfall map of Britain for December 2015, comparing actual rainfall with the average expected for the month of December

Monthly rainfall data was obtained for Trawsfynydd in North Wales, covering the period from 1950 to 2010. This very complete data set was collected by staff of a hydro-electric power station, who provided access to the original written records of the weather station observations. This data is expected to be accurate.

The data was entered into a spreadsheet and the graph in figure 183 was plotted. We see a wide variation in rainfall from month to month, and from year to year. Extreme rainfall months occur randomly. However, there does seem to be a slight increase in mean rainfall since a drier period around the end of the 1960's. Again, there may be cause for concern if rainfall continues to rise at a similar or increasing rate in future years.

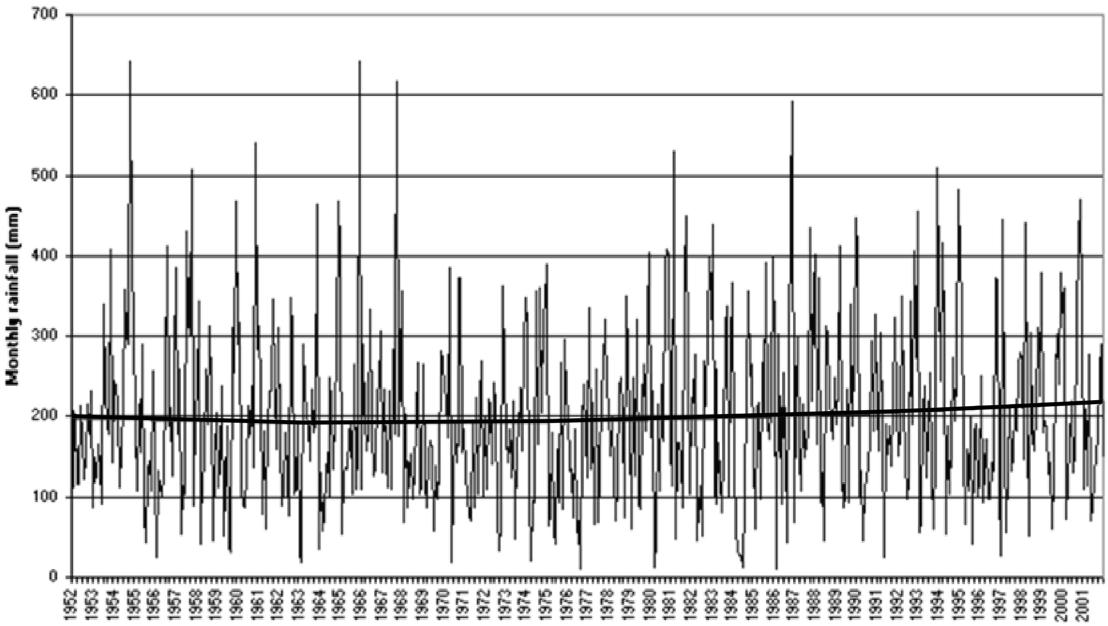


Figure 183: Monthly rainfall for Trawsfynydd, North Wales, for the period 1950 – 2010. A trend line has been added, showing a slight increase in mean rainfall in recent years

We might now look more widely at variations in world rainfall patterns. This becomes more problematic, as no complete data sets are easily available. We have to fall back on a computer model of predicted rainfall changes made by scientists of the Intergovernmental Panel on Climate Change. Although produced by experts, the data is not based on actual observation but is a mathematical best estimate after making various assumptions about global warming. It might be argued that the Intergovernmental Panel on Climate Change is a political group set up to warn people of the dangers of climate change, so may not be neutral. As a consequence, the data should be treated with caution.

The model shown in Figure 184 predicts increased rainfall in temperate and equatorial areas, with slightly reduced rainfall in the tropics. The change is stated in terms of standard deviation. The absolute increase or decrease in rainfall is not easy to determine from this map, but is likely to be in the order of a 20% - 30% increase in rainfall in the northern hemisphere over the coming 50 years. The predictions are consistent with other evidence, so seem to be qualitatively plausible.

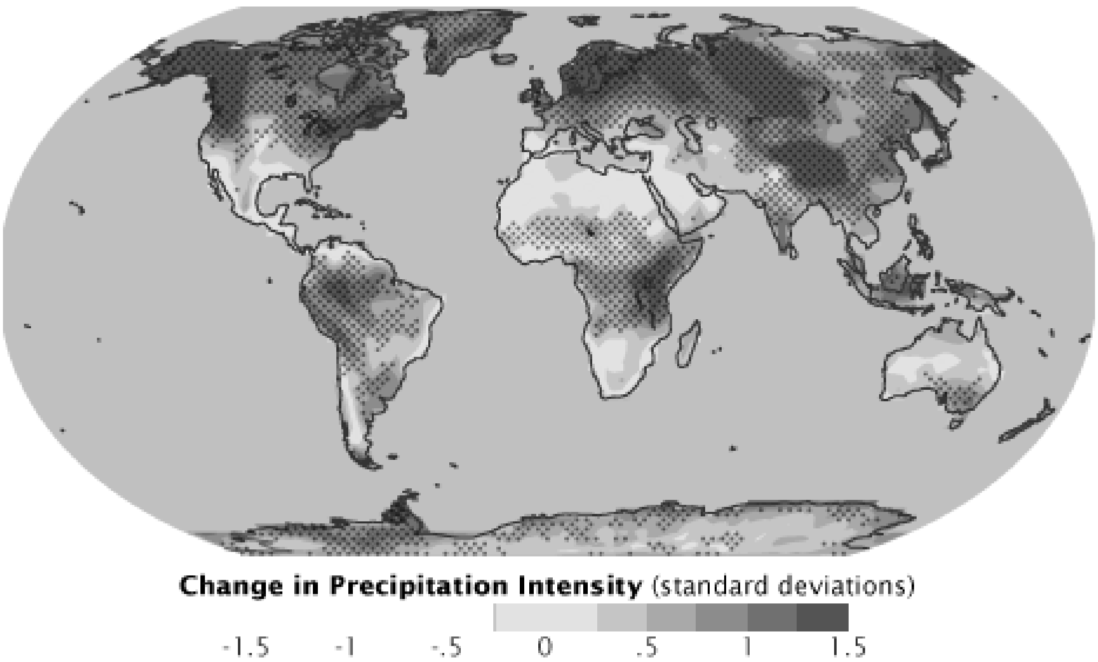


Figure 184: Computer model for likely changes in rainfall

Evidence seems to suggest that climate change is occurring, although the effect so far is not very large. Let us now consider the question of whether this climate change is a natural phenomenon or the result of human activity.

Scientists have developed a convincing theory that gases such as carbon dioxide are able to trap heat in the Earth's atmosphere, leading to global warming. This is termed the ***greenhouse effect***. We might investigate whether carbon dioxide levels in the atmosphere have increased over the past couple of centuries as a result of increased industrialisation.

Figure 185 compiled by the Intergovernmental Panel on Climate Change shows large increases in the past 200 years in the amounts of three main greenhouse gases: carbon dioxide, methane and nitrous oxide. This data has been collected mainly by analysis of gas trapped in glacier ice samples of known ages. The data has been obtained by expert scientists who would be expected to have reported their findings accurately, so can be treated as reliable.

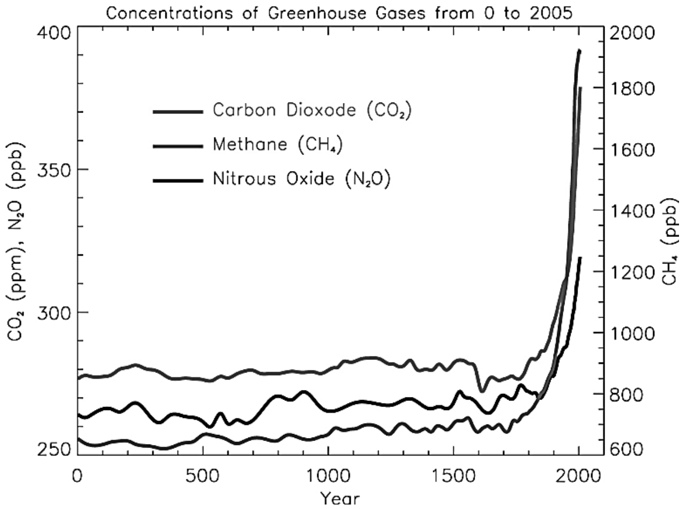


Figure 185:

Concentrations of the principal greenhouse gases in the atmosphere over the past 2000 years

If we accept the link between greenhouse gases and climate change, it then becomes necessary to identify the causes of increased concentrations of the greenhouse gases so that corrective action can be taken.

It is known that carbon dioxide is produced by burning fossil fuels such as coal, oil or gas. However, the CO_2 concentration is also affected by vegetation, particularly forest, which can extract carbon dioxide from the air during photosynthesis and retain this as biomass.

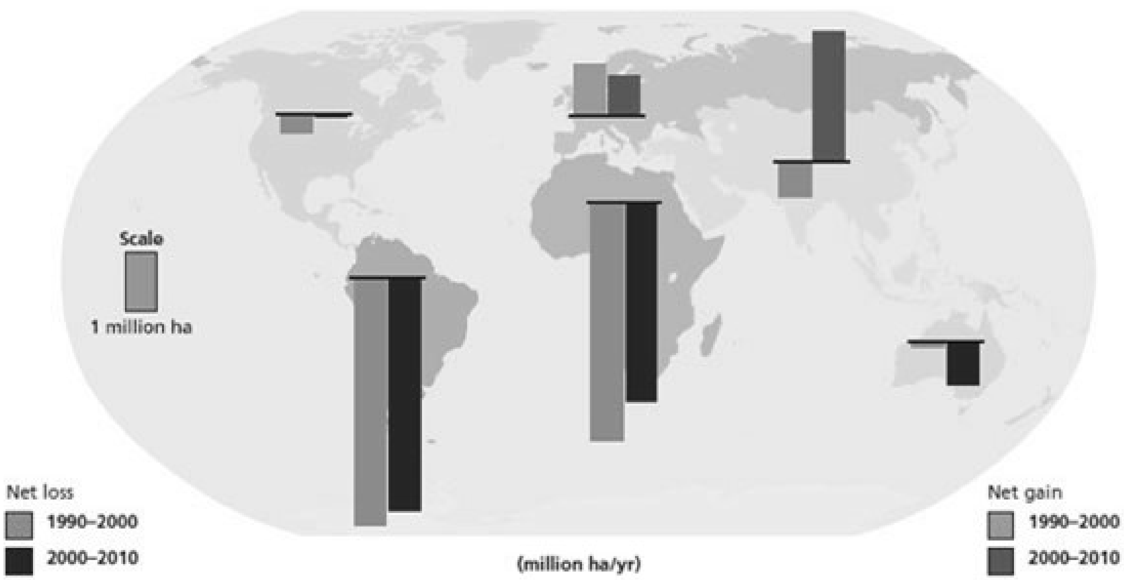


Figure 186: Changes in the area of forest in each continent over the period 1990 - 2010

Figure 186 is a map showing changes in the area of forest. This was produced by the Food and Agriculture Organisation of the United Nations, which is an organisation with a good reputation for reliability. Data is likely to have been compiled by experts, so we may conclude that the map is accurate. Extensive loss of forest is continuing in Africa and South America, though this is offset to some extent by an increase in the area of forest in Asia and Europe.

We might now consider the effects of climate change on people around the world. The most urgent problem for a number of coastal communities is sea level rise. Figure 187 has been compiled by scientists of the Commonwealth Scientific and Industrial Research Organisation and the University of Colorado, USA, who are in agreement about recent trends in sea level. We might expect this data to be accurate.

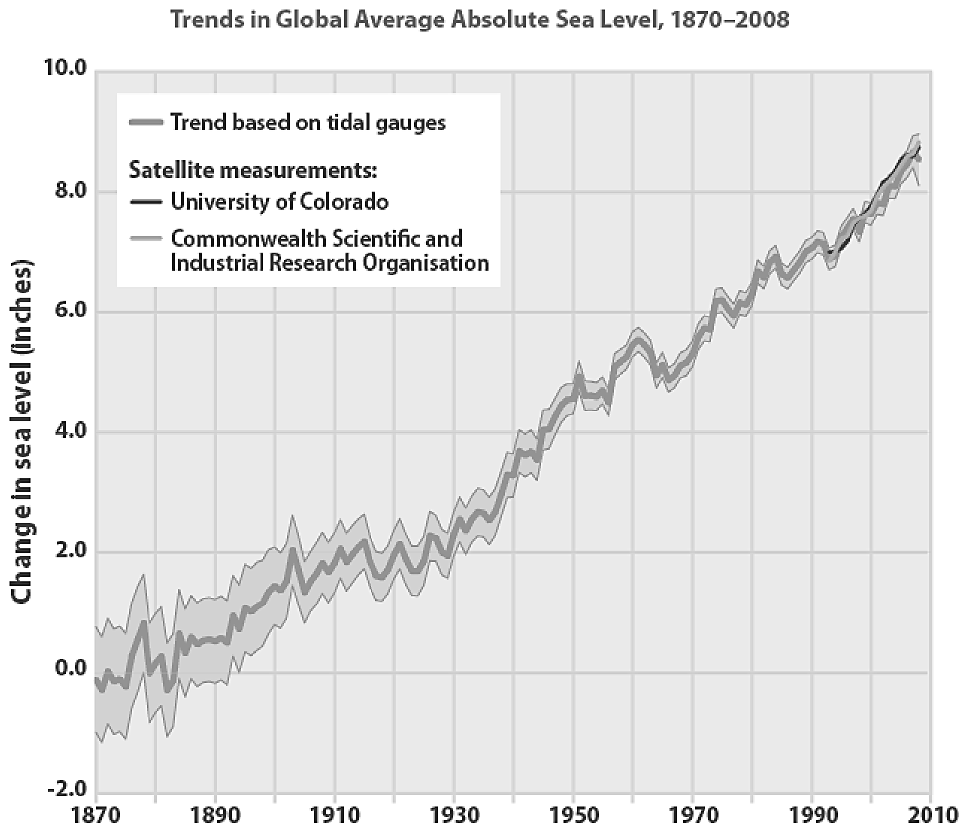


Figure 187: Changes in sea level from 1870 to 2010

Change in sea level over the period is about 9 inches (22 cm). This is not large, but could become an increasing cause for concern if the trend continues.

We should, at this point, look for evidence linking sea level rise to global warming. Scientists consider that sea level is rising due to the melting of ice from the polar ice caps, and to a lesser extent from mountain glaciers. Maps in figure 188 compare the extent of the September ice cap over the northern polar region for the years 1980 and 2007. These maps were compiled from satellite images by the US National Snow and Ice Data Center, so are likely to be accurate.

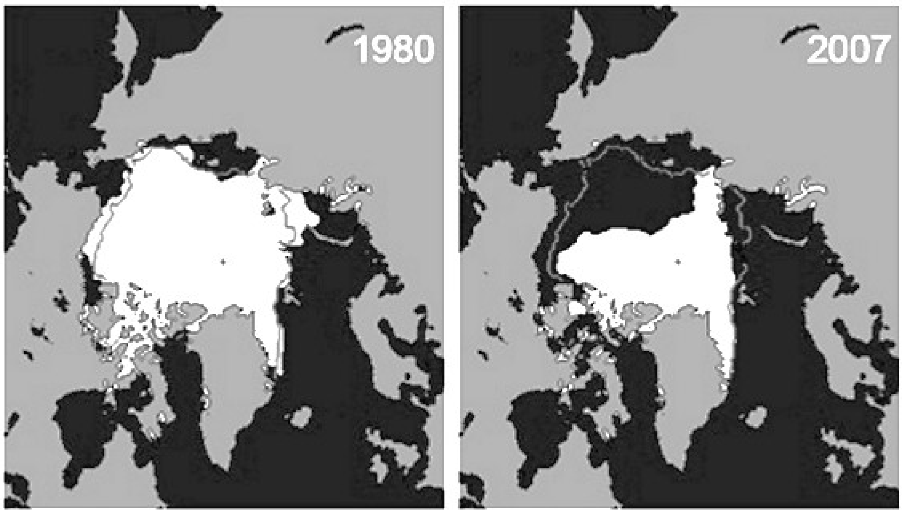


Figure 188: Extent of September ice cap in the northern polar region

Data is only shown for two particular years. It seems likely that a long term loss of ice is occurring, but we might wish to examine data for other years to confirm this.

Another danger which might be associated with climate change is an increase in the number of damaging tropical storms. Figure 189 is produced from data collected by the US National Hurricane Center. With satellite observation, there have been improvements in the detection of very short duration hurricanes and cyclones. These short lived events have been omitted so that data from earlier decades is properly comparable. There is a slight upwards trend in the number of hurricane and cyclone events, but with considerable variation from year to year. As with extreme rainfall events, the effects of climate change are not yet great, but could become a cause for concern if the upward trend continues and escalates.

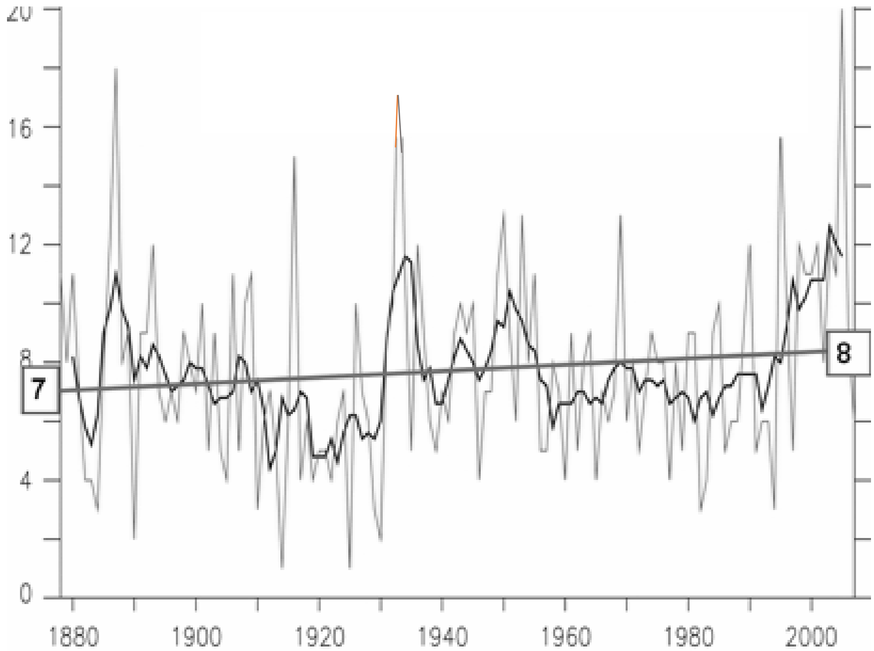


Figure 189: Number of hurricanes and cyclones recorded for the years 1880 to 2010

We might now investigate the occurrence of droughts. Drought is caused by a difference between the water demand and the water available. Whilst the likelihood of a drought is clearly affected by reduced rainfall, it can be made worse by an increasing water demand from homes, agriculture and industry. Water demand per person generally increases as countries become more technologically developed.

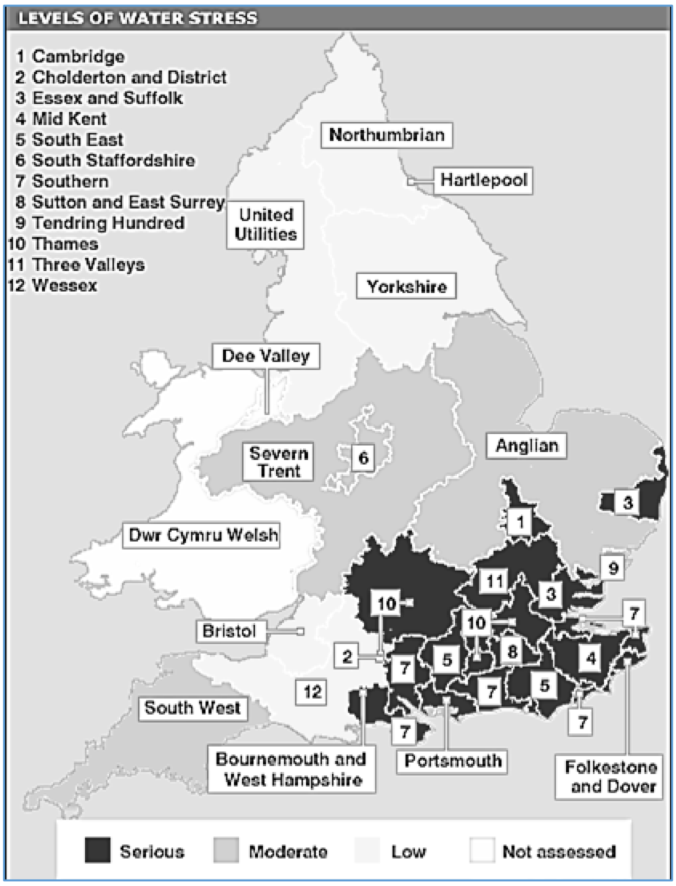


Figure 190:

Areas of England where serious water shortages were experienced during drought year of 2006

Figure 190 is a map compiled by the Environment Agency to identify areas of England where water supply is insufficient to meet demand during years with low rainfall. The exact meaning of the terms 'moderate' and 'serious' is not defined, but we might assume that urgent action is needed in these areas, either to reduce water demand, collect additional water from rainfall, or bring in extra water supplies from other regions.

To summarise the data on climate change, we might conclude that:

- Temperatures are rising, but not very fast.
- Mean rainfall is slightly increasing in tropical and temperate areas, and slightly falling in tropical areas.
- Catastrophic weather events such as floods and hurricanes are very variable from year to year, and there is no strong evidence that current weather events are more extreme than those which have occurred in the past. However, continued global temperature rise is likely to increase the risk from extreme weather.

- There is clear evidence that atmospheric concentrations of carbon dioxide, methane and nitrous oxide are increasing. Modelling can show a link between concentrations of these gases and temperature rise through the *greenhouse effect* theory.
- Increases in carbon dioxide in the atmosphere may be due to burning of fossil fuels, and also to extensive loss of forest. Reduction in CO_2 production from fossil fuel burning seems sensible, particularly since alternative energy sources are available. Forestry regeneration can have a beneficial effect in regulating CO_2 concentrations.
- There is clear evidence of a small but significant rise in sea level. This can be convincingly linked to evidence of ice cap reduction.
- Water deficits occur in various regions. These can be ascribed not just to climate change but also to increased water demand from agriculture, industry and the population.

None of these issues are simple, but acquiring reliable data and interpreting it effectively can allow stronger evidence-based decisions to be made.

In the following sections, we move on to examine some specialist types of graph which are appropriate for displaying particular data sets obtained during vocational course activities.

Cyclic graphs

Some data sets contain a pattern which repeats at intervals. This might, for example, be a daily or yearly cycle. A longer term trend may be superimposed on the repeating pattern. As an example, let us consider how we might make sales predictions when preparing a business plan for a new restaurant in a small town in Snowdonia.

Most businesses are to some extent seasonal. A restaurant would expect to have regular customers who live or work locally, but also many visitors who are on holiday in the area. Peak periods of the year might be Christmas, Easter and the summer holiday period.

We will begin by predicting the relative sales in each of the twelve months as a comparison to the busiest month of July. Experience from similar restaurant businesses might suggest:

January	0.5	July	1.0
February	0.3	August	0.9
March	0.6	September	0.8
April	0.8	October	0.5
May	0.7	November	0.3
June	0.8	December	0.6

A model can be produced with an Excel spreadsheet. We will set up the monthly sales values as a **lookup table**, so that the values can be accessed when required for the sales calculations.

We will begin by assuming that sales will be £20, 000 in July of the first year. This base value allows sales for other months to be calculated.

	A	B	C	D	E	F	G	H	I
1	Month number	Month	Sales		Month		base	fraction	sales
2	1	Jan	0.5		1	Jan	20000	0.5	10000
3	2	Feb	0.3		2	Feb	20000	0.3	6000
4	3	Mar	0.6		3	Mar	20000	0.6	12000
5	4	Apr	0.8		4	Apr	20000	0.8	16000
6	5	May	0.7		5	May	20000	0.7	14000
7	6	Jun	0.8		6	Jun	20000	0.8	16000
8	7	Jul	1.0		7	Jul	20000	1	20000
9	8	Aug	0.9		8	Aug	20000	0.9	18000
10	9	Sep	0.8		9	Sep	20000	0.8	16000
11	10	Oct	0.5		10	Oct	20000	0.5	10000
12	11	Nov	0.3		11	Nov	20000	0.3	6000
13	12	Dec	0.6		12	Dec	20000	0.6	12000
14					13	Jan	20000	0.5	10000
15					14	Feb	20000	0.3	6000
16					15	Mar	20000	0.6	12000

Figure 191: Predicted sales for the restaurant, based on relative monthly sales values

Using the predicted income for July of the first year as a base value, we can produce a graph of annual sales, allowing for variations from month to month.

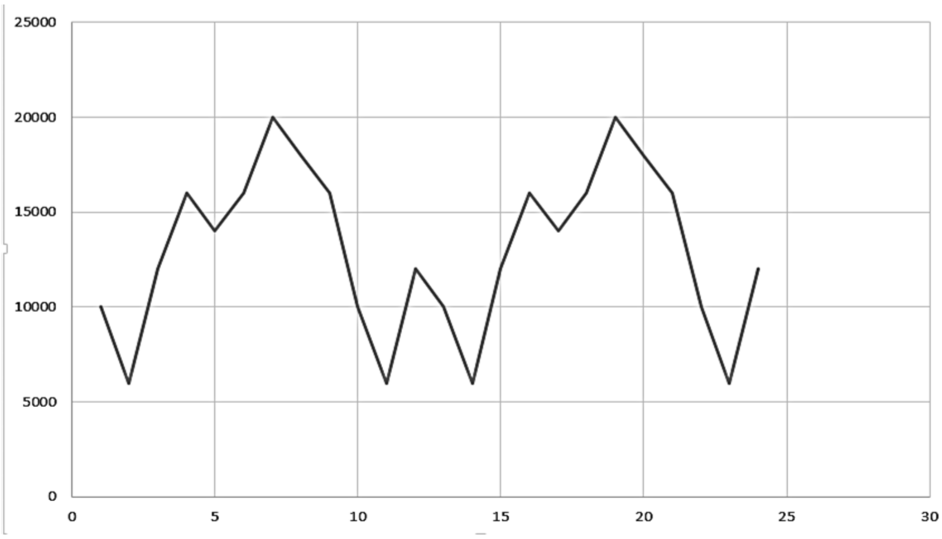


Figure 192: Predicted sales based on a constant annual sales total

We might now make an assumption that business will increase for the first few years at a rate of 24% year-on-year growth as the restaurant becomes well known. This will increase the takings by 2% per month. This effect can be superimposed on the seasonal trend. Extending the model for five years gives a graph of sales.

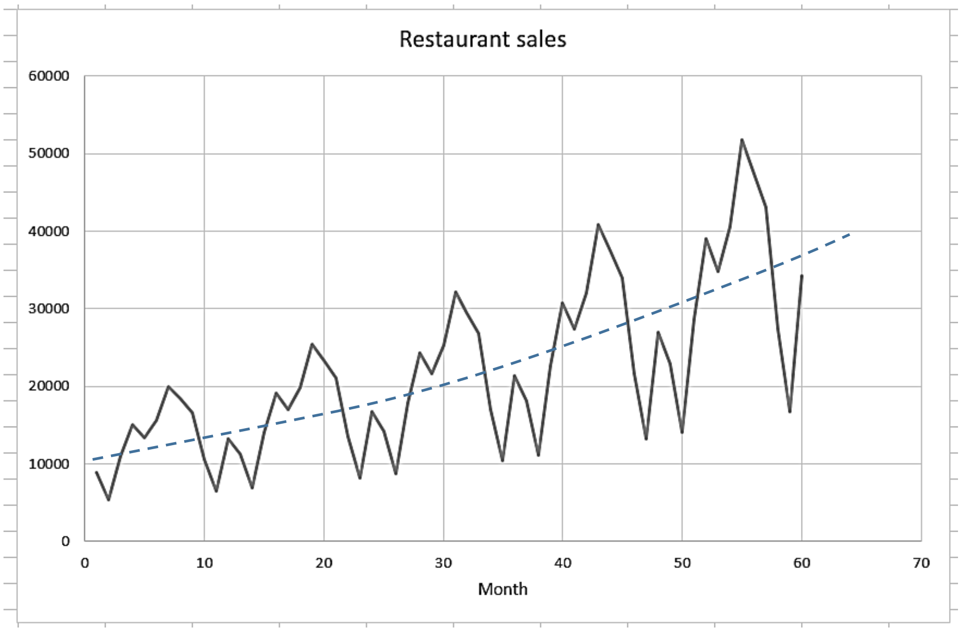


Figure 193: Predicted sales based on an increasing annual sales total

By use of a spreadsheet, it is easy for business studies students to experiment in this way with financial forecasts when developing business plans.

Tides

Another cyclic process which we will examine is the prediction of tidal heights. Tides are primarily influenced by the moon. The moon exerts a gravitational attraction on ocean waters, causing a tidal bulge on the side of the Earth nearest to the moon. However, the rotation of the earth-moon system also leads to a concentration of ocean waters on the opposite side of the earth. This region is furthest from the moon where the gravitational effect is lowest and centrifugal force can increase the tidal height.

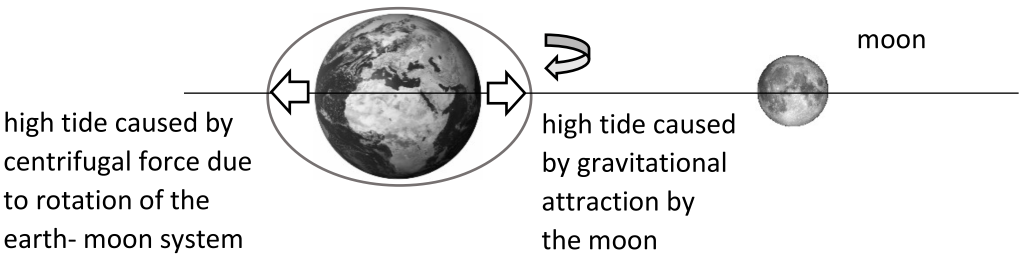


Figure 194: Tidal influence of the moon

Another smaller influence on tides is the sun. This has a huge mass, although it is vastly further away from the earth then the moon. The effect of the sun on tides is not constant. The moon orbits around the Earth approximately every four weeks. Every fortnight, the sun and moon lie in approximately the same line, so that tidal effects are increased. This produces particularly high **spring tides**.

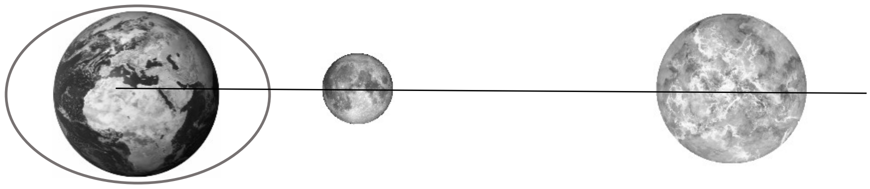


Figure 195: Spring tides produced by a sun-moon alignment

A week after the spring tides, the moon and sun will be positioned at right angles when observed from the Earth. The cumulative tidal effect is reduced, and lower **neap tides** are produced.

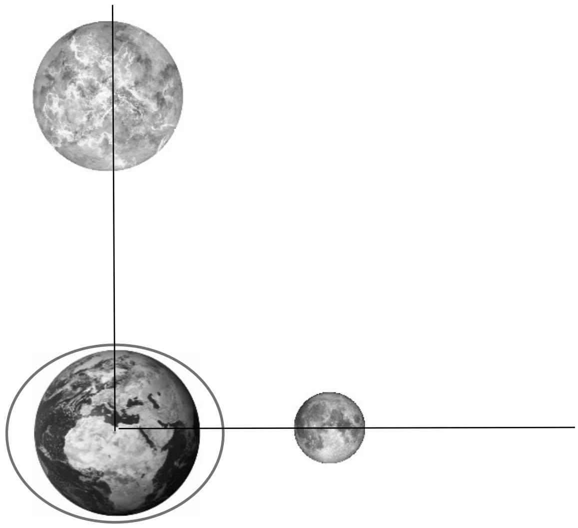


Figure 196: Neap tides produced by the sun and moon aligned at right angles relative to the earth

A further effect is produced by the angle of the Earth's axis relative to the orbit of the moon. This angle is known as the **lunar declination**, and changes over a period of weeks. For a lunar declination in the same hemisphere, tidal height would be highest when the moon is on the adjacent side of the Earth.

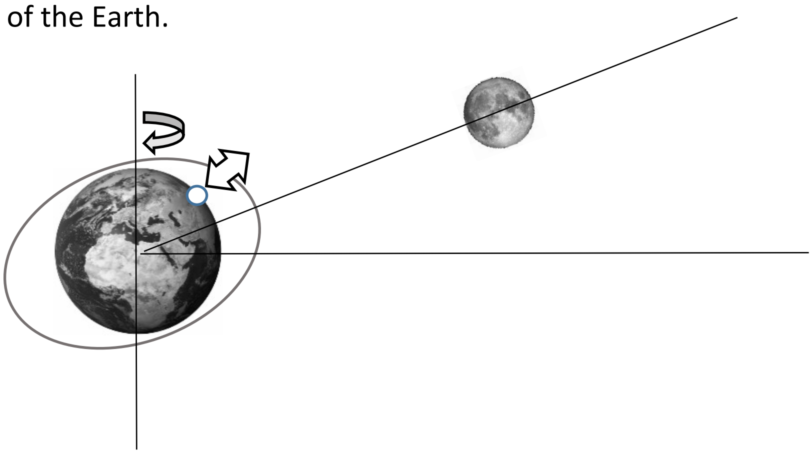


Figure 197: Lunar declination producing a higher tidal peak in the region facing the moon

The high tide event 12 hours later will have a reduced height. The location is now on the opposite side of the earth and further from the peak of the tidal bulge.

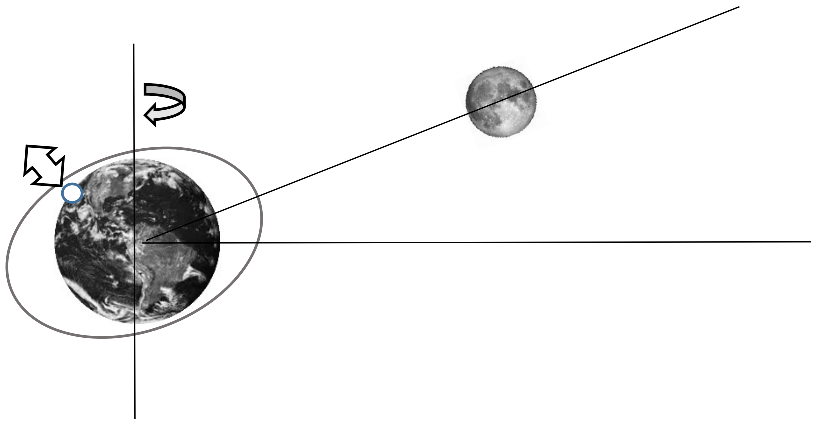


Figure 198: Lunar declination producing a lower tidal peak in the region facing away from the moon

The complex interaction of these factors can be seen in the example tidal graph for Barmouth, North Wales (figure 199). High and low tide heights vary systematically over the spring-neap tidal cycle. There is also a variation between the two high tides each day.

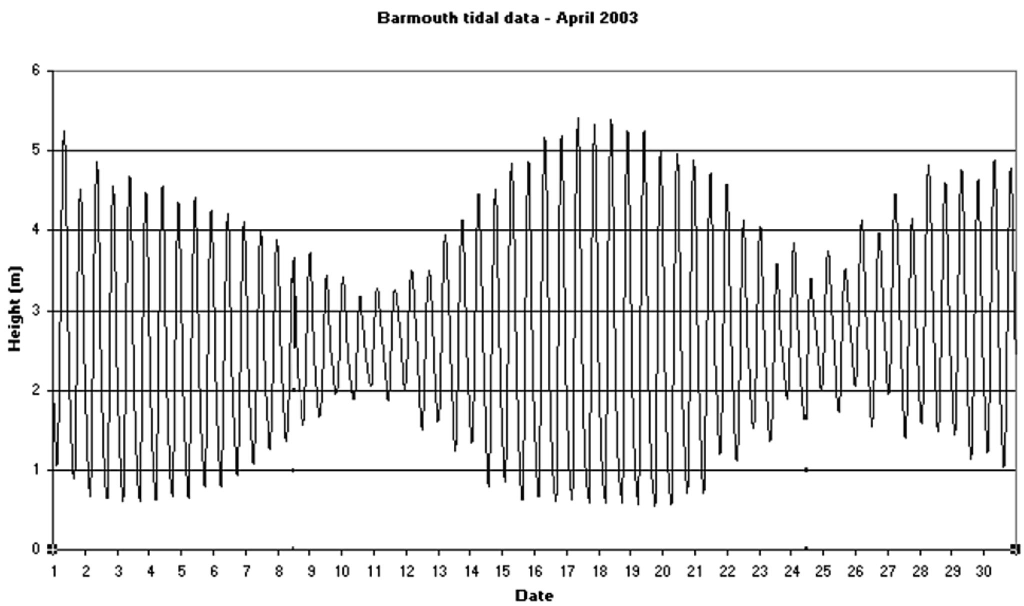


Figure 199: Typical variation in tidal height during a month

Tidal prediction involves a complex calculation, but is based on adding together a series of cosine functions of the form:

$$H = A\cos(\omega t + p)$$

where:

H is the contribution to the total tidal height,
A is the maximum amplitude of this function,
ω is the angular velocity which determines the frequency of the cosine curve
t is the time
p is a phase angle which is used to coordinate the functions relative to one another.

Three functions have been set up in a spreadsheet to demonstrate the principle of the method, though many more would be included for an accurate tidal prediction.

	A	B	C	D	E
1	Amplitude	300	120	40	
2	Frequency	1	0.95	0.6	
3	Phase	0	60	30	
4					tidal height
5	0	300.00	60.00	34.64	394.64
6	20	281.91	22.90	29.73	334.53
7	40	229.81	-16.70	23.51	236.62
8	60	150.00	-54.48	16.27	111.79
9	80	52.09	-86.32	8.32	-25.91

	A	B	C	D	E
1	Amplitude	300	120	40	
2	Frequency	1	0.95	0.6	
3	Phase	0	60	30	
4					
5	0	=$B$1*COS((A5*$B$2+$B$3)*PI()/180)	=$C$1*COS((A5*$C$2+$C$3)*PI()/180)	=$D$1*COS((A5*$D$2+$D$3)*PI()/180)	=B5+C5+D5
6	20	=$B$1*COS((A6*$B$2+$B$3)*PI()/180)	=$C$1*COS((A6*$C$2+$C$3)*PI()/180)	=$D$1*COS((A6*$D$2+$D$3)*PI()/180)	=B6+C6+D6
7	40	=$B$1*COS((A7*$B$2+$B$3)*PI()/180)	=$C$1*COS((A7*$C$2+$C$3)*PI()/180)	=$D$1*COS((A7*$D$2+$D$3)*PI()/180)	=B7+C7+D7
8	60	=$B$1*COS((A8*$B$2+$B$3)*PI()/180)	=$C$1*COS((A8*$C$2+$C$3)*PI()/180)	=$D$1*COS((A8*$D$2+$D$3)*PI()/180)	=B8+C8+D8

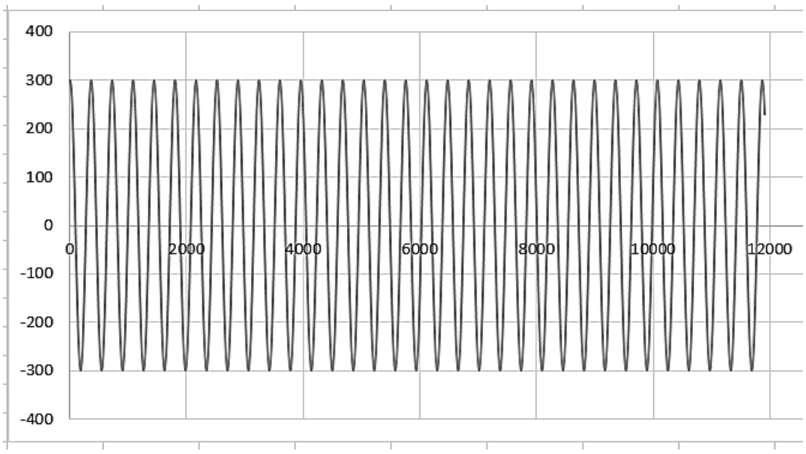
Figure 200: Spreadsheet to demonstrate tidal calculation

The lower display in figure 200 illustrates formulae for calculating three cosine functions, which are then added in column E to produce the tidal graph output.

If values are first assigned only to cells B1-B3, leaving cells C1–D3 blank, then a basic cosine curve is produced. This would represent the tidal effect of only the moon.

Figure 201:

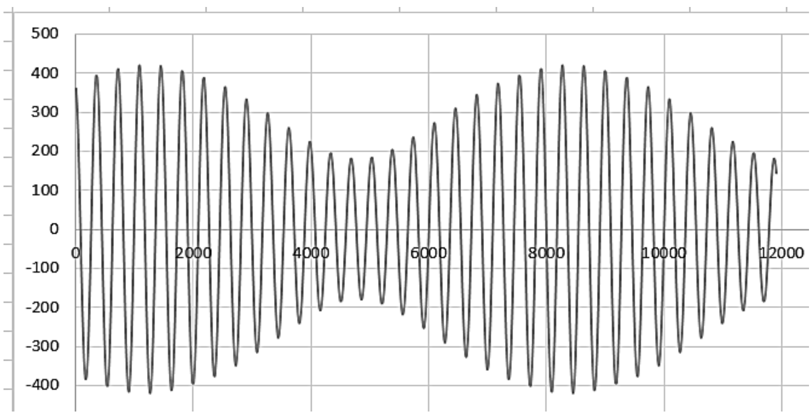
Tidal effect of the moon alone



If values are now added for cells C1-3, the effect of the sun in producing spring and neap tides is added.

Figure 202:

Effects of the moon and sun in producing spring and neap tides



Values in cells D1-3 then add the effects of a lunar declination, producing a variation between successive tides.

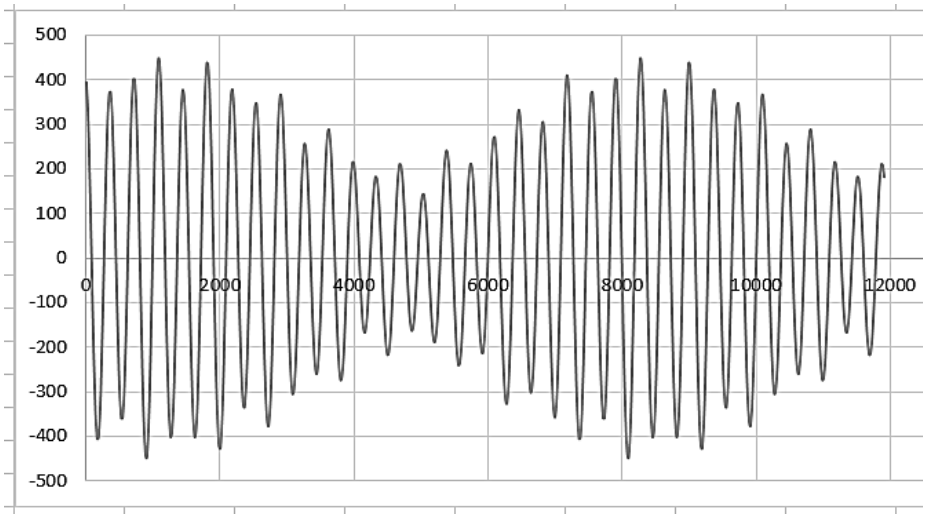


Figure 203: Tidal graph produced from the sum of three cosine functions

Tidal prediction depends on determining the correct values for amplitude, angular velocity and phase to specify the cosine functions for a particular location. Methods for obtaining these parameters using astronomical observations were determined in the 1920's by Arthur Doodson, a Fellow of the Royal Society.

A further complicating factor in tidal calculations is that the Earth is not exactly at the centre of circular orbits of the sun and moon. At certain times the moon and sun are closer or further from the Earth.

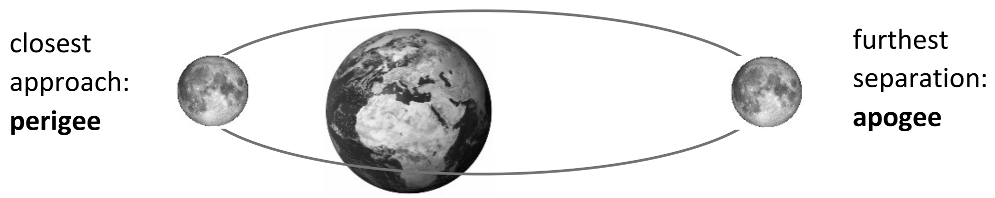


Figure 204: Elliptical orbit of the moon around the earth

This change in separation causes a change in the amplitude for the tidal effect. The tidal amplitude parameter A does itself vary as a cosine function:

$$A = A_0[1 + \mathrm{A}_v \cos(\omega t + p_v)]$$

For a full model, the functions which are added during the calculation should have the form:

$$A = A_0[1 + \mathrm{A}_v \cos(\omega t + p_v)] \ .\cos(\omega t + p)$$

The geometry of neighbouring land masses can also affect tidal heights. For example, the English Channel has a complex pattern of double high tides, due to constriction of tidal flows in the narrow and shallow sea area between England and France. Tidal predictions for any location may therefore depend on both ***astronomical*** and ***geographical*** factors.

Circular diagrams

Graphs normally use a system of horizontal and vertical Cartesian coordinates **x** and **y** to represent data points. However, it is sometimes convenient to use polar coordinates, where data points are represented by an angle **ϴ** and radial distance **r** from the origin:

Figure 205: (left) Cartesian coordinates and (right) polar coordinates

A simple use of polar coordinates is to represent a number of related factors, so that they can be easily visualised and compared. As an example, we will look at research into Welsh-English bilingualism of college students. The objective of the project is to identify the different ways in which students make use of the Welsh language in their everyday lives, and to identify opportunities for further language development within particular student groups.

Students are asked to complete a questionnaire, in which they are presented with a series of statements such as:

I speak Welsh with friends socially

and are then asked to give a score between 0 and 10 on a Likert scale:

0	1	2	3	4	5	6	7	8	9	10
disagree completely			disagree slightly		neutral		agree slightly			agree completely

Figure 206: Likert opinion scale

Results for individual students can be displayed in a polar diagram, and results can be grouped for the students studying particular courses.

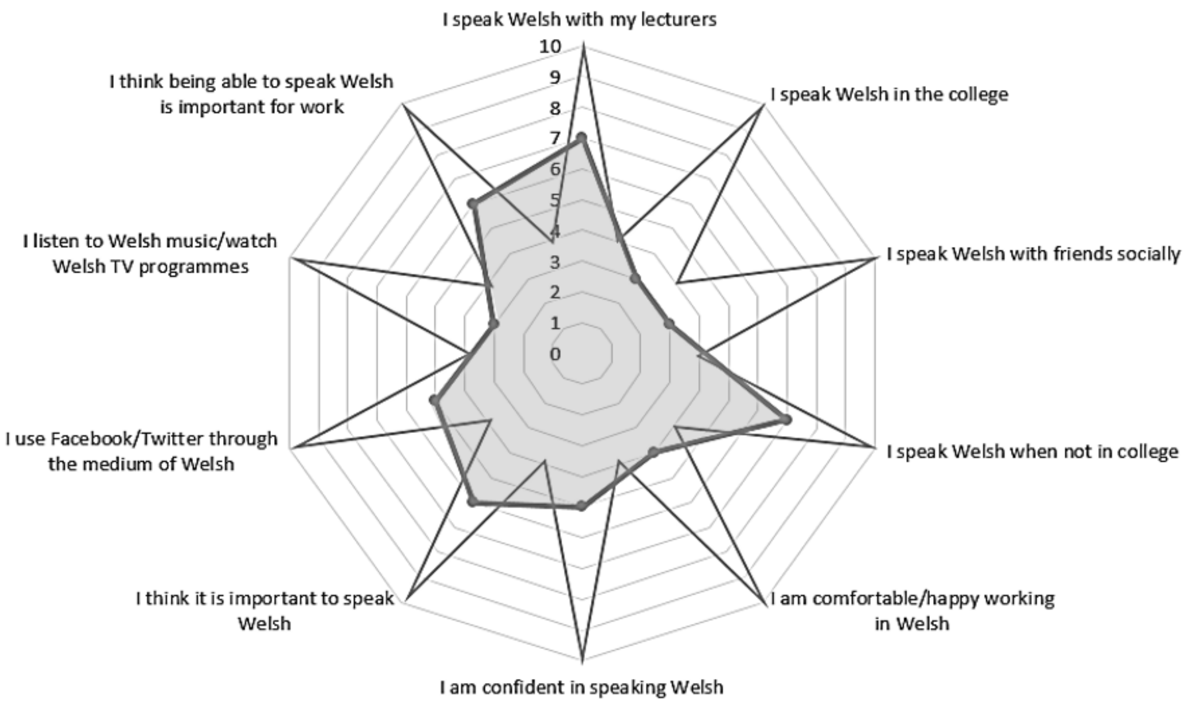


Figure 207: Circular diagram of Welsh language use by students

In this way, opportunities can be identified for building on existing use of Welsh, e.g.

I speak Welsh with my lecturers

and for introducing students to new activities which use the Welsh language, e.g.

I listen to Welsh music/watch Welsh TV programmes

In the next example, a circular diagram is suitable because the data displayed is directional. Angles in the polar diagram can represent compass bearings.

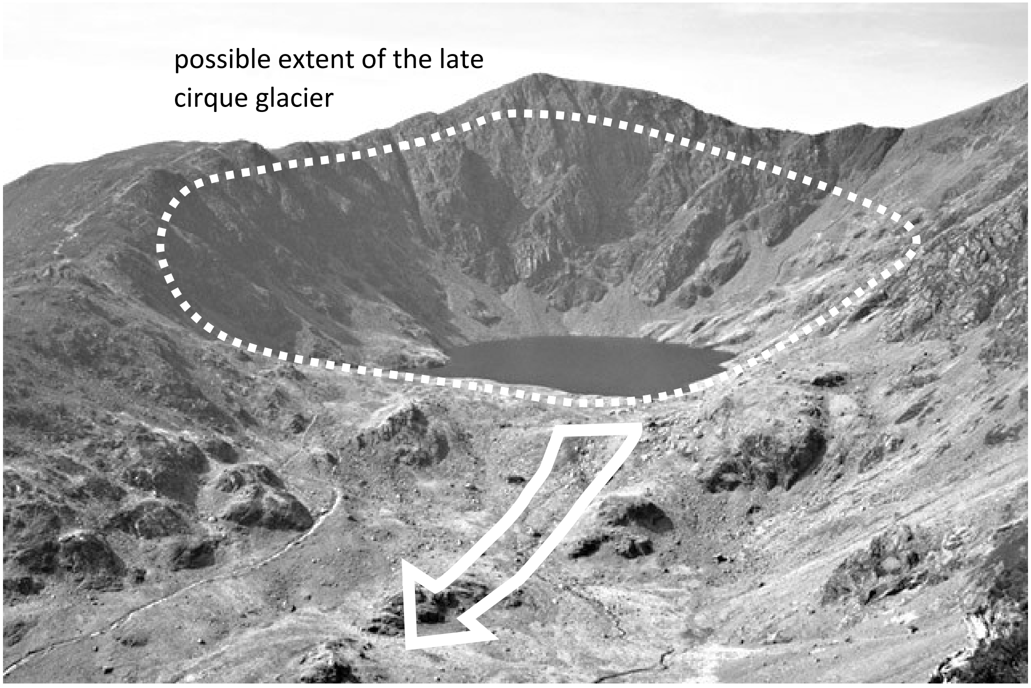
Till fabric analysis

Upland areas of Britain were subject to extensive glaciation during the Ice Age, and a study of glacial landforms and glacial deposits is an important aspect of geomorphology.

Moving ice sheets carry rock debris ranging in size from boulders to clay particles. This material can be deposited as moraine below and at the edges of a moving ice sheet, or at the limit of a glacier where melting occurs. A study of an area moraine can give useful information about the flow direction of the ice which deposited the material. Fragments of rock become oriented within the moving ice, so that their longest axes lie in the direction of ice flow. This alignment can be preserved when the rock fragments are deposited as moraine.

In this example, we will look at the results of a till fabric analysis study by geology students of the Cwm Cau glacial cirque basin on the mountain of Cader Idris in North Wales.

Glacial deposits preserved in Cwm Cau belong to the Devensian period which occurred at the end of the Ice Age. During the main Devensian glaciation, this valley was the source of a large glacier which flowed out to join the larger Tal y Llyn glacier in the valley below.



direction of the main valley glacier

Figure 208: Glacier ice in Cwm Cau

A warm interglacial period followed the main glaciation, with ice melting from the mountains. Glacial conditions returned in the late Devensian, allowing a cirque glacier to develop at the head of Cwm Cau. Ice accumulating beneath the cliffs at the head of the cirque moved downwards in a rotational motion, depositing moraine against the rock step at the mouth of the cirque, and releasing rock material from the snout of the glacier as melting occurred.

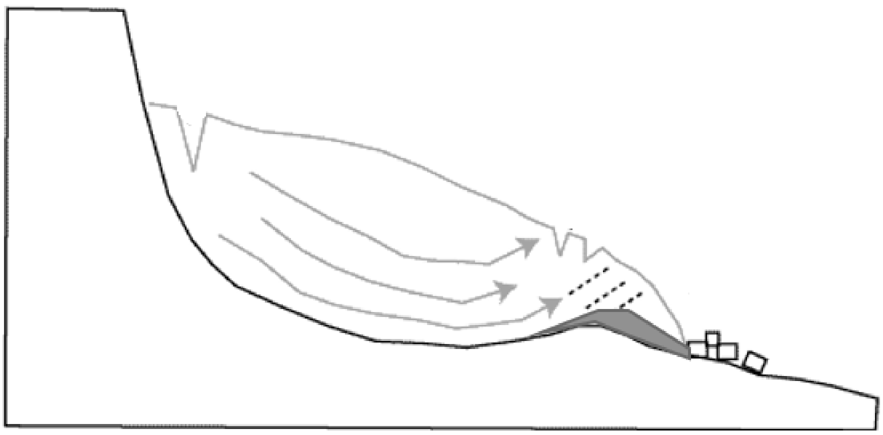


Figure 209: Ice movement in a cirque glacier

Data was collected at two locations in Cwm Cau: in the moraine ridge at the mouth of the cirque basin, and lower down the valley in moraine exposed in the bank of the Nant Cader stream.

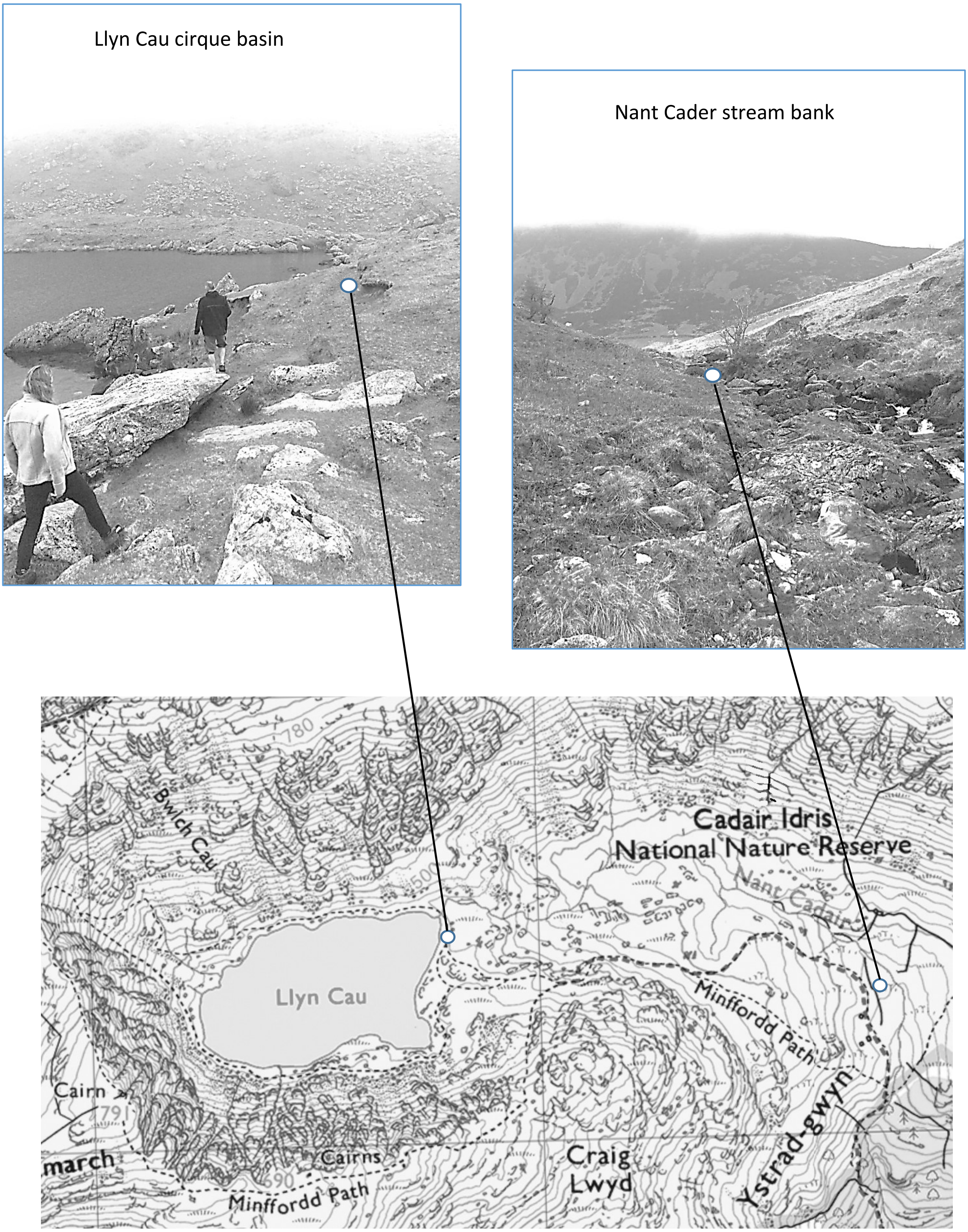
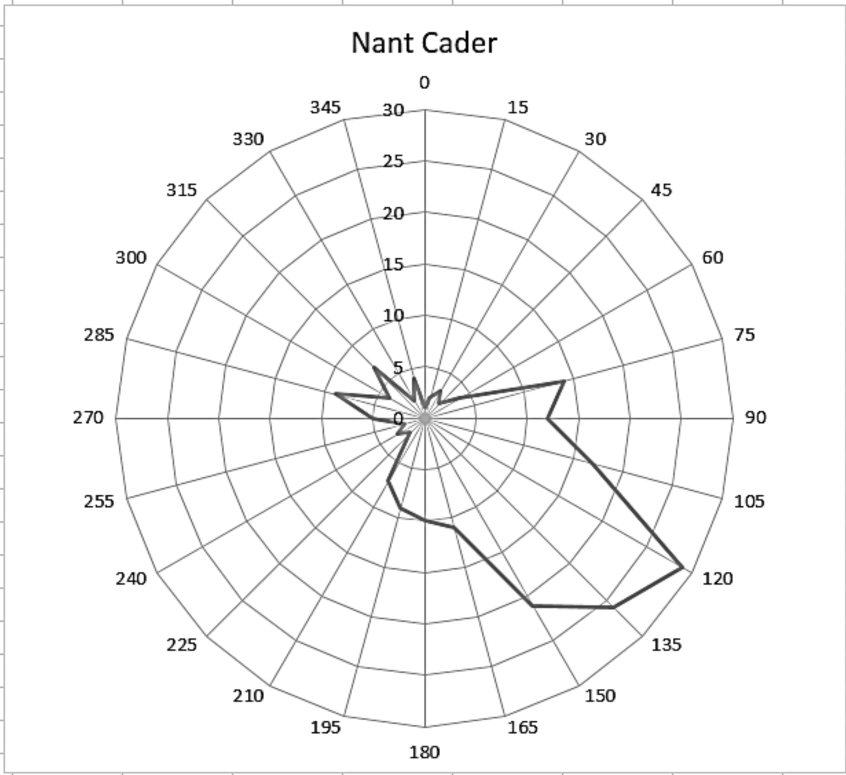


Figure 210: Moraine sampling sites in Cwm Cau

sector	count
0	1
15	2
30	3
45	2
60	4
75	14
90	12
105	17
120	29
135	26
150	21
165	11
180	10
195	9
210	7
225	2
240	3
255	2
270	5
285	9
300	4
315	7
330	2
345	4



sector	count
0	1
15	2
30	3
45	5
60	9
75	12
90	11
105	7
120	3
135	4
150	1
165	2
180	3
195	2
210	4
225	7
240	15
255	17
270	29
285	22
300	18
315	8
330	5
345	4

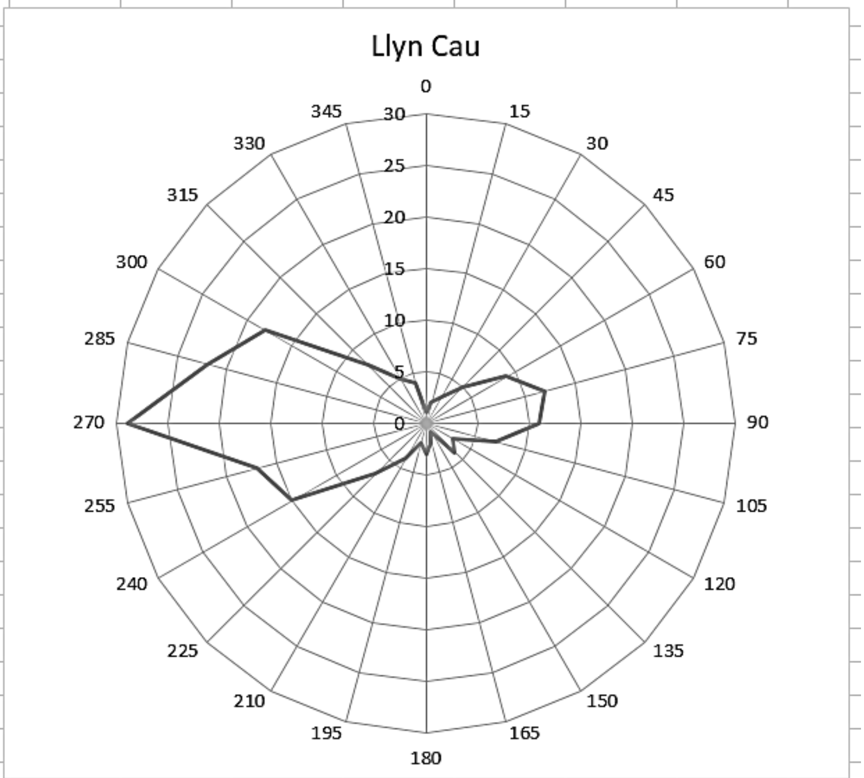


Figure 211: Till fabric analysis results for the Cwm Cau sampling sites

At each of the sites, a number of rock fragments embedded within the clay matrix of the moraine were examined, and a compass was used to measure the downwards dip directions of their long axes. Results are plotted as circular diagrams in figure 211, with the data points recording the total numbers of samples falling within each 15 degree sector.

The circular plot for the Nant Cader stream bank exposure illustrates the downwards flow of the glacier from the upland valley, and represents ground moraine deposited at the base of the moving ice.

The circular plot for Llyn Cau is interesting in showing a predominance of dips of rock fragments back towards the cirque basin. This is consistent with the material being deposited at the base of the ice flow as it was rising over the rock lip of the cirque basin in a rotational motion.

Alternating current

Polar graphs are an interesting way to display the characteristics of alternating currents when carrying out experiments in electronics or electrical physics. AC supplies may have the same frequency, such as 50Hz, but have different peak voltages and phase angles. If these alternating currents are combined, it is found that the resultant is also an alternating current, but with a different peak voltage and phase angle to the two constituent wave forms.

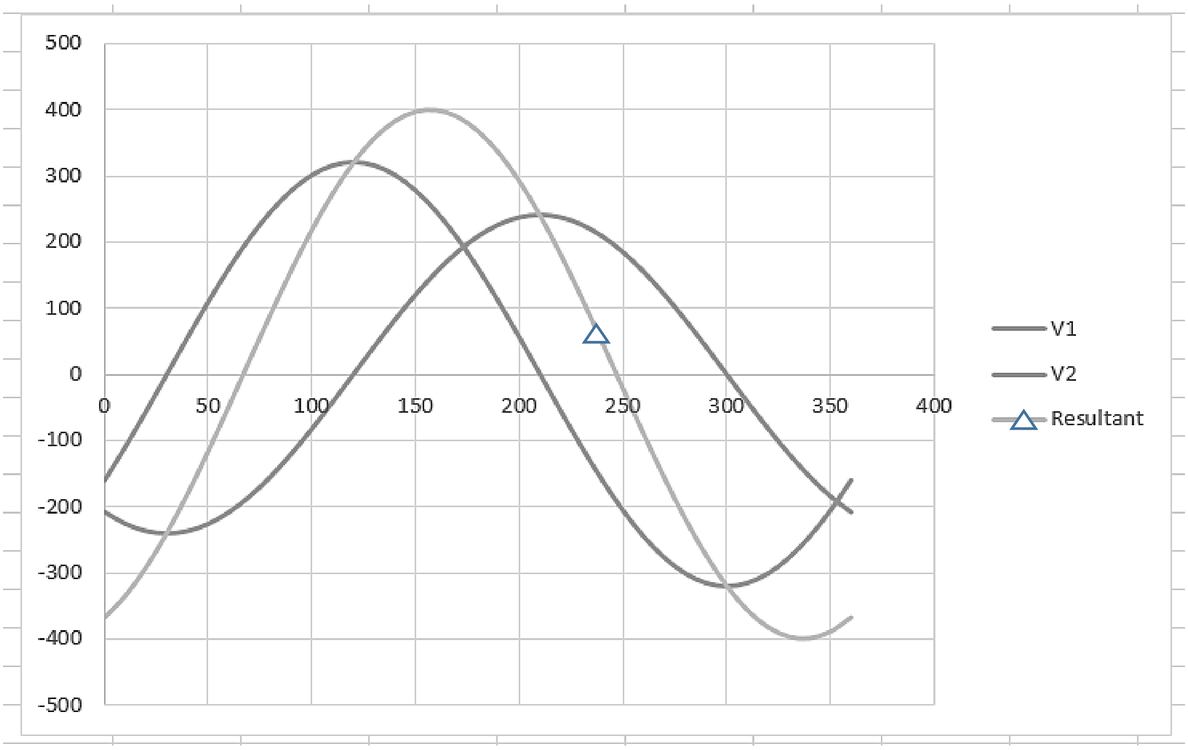


Figure 212: Combination of two alternating currents

A spreadsheet can be set up to input the phase angles and peak voltages for two sine waves, then add the wave forms to produce a resultant.

	A	B	C	D	E	F
1		phase 1	0	phase 2	120	
2		voltage 1	340	voltage 2	155	
3						
4	angle	V1 degrees	V1 volts	V2 degrees	V2 volts	resultant
5	0	0	0	-120	-134.234	-134.233938
6	10	10	59.04038	-110	-145.652	-86.6119758
7	20	20	116.2868	-100	-152.645	-36.358353
8	30	30	170	-90	-155	15
9	40	40	218.5478	-80	-152.645	65.9025856
10	50	50	260.4551	-70	-145.652	114.802754

	A	B	C	D	E	F
1		phase 1	0	phase 2	120	
2		voltage 1	340	voltage 2	155	
3						
4	angle	V1 degrees	V1 volts	V2 degrees	V2 volts	resultant
5	0	=-$C$1	=$C$2*SIN(B5*PI()/180)	=-$E$1	=$E$2*SIN(D5*PI()/180)	=C5+E5
6	=A5+10	=B5+10	=$C$2*SIN(B6*PI()/180)	=D5+10	=$E$2*SIN(D6*PI()/180)	=C6+E6
7	=A6+10	=B6+10	=$C$2*SIN(B7*PI()/180)	=D6+10	=$E$2*SIN(D7*PI()/180)	=C7+E7
8	=A7+10	=B7+10	=$C$2*SIN(B8*PI()/180)	=D7+10	=$E$2*SIN(D8*PI()/180)	=C8+E8
9	=A8+10	=B8+10	=$C$2*SIN(B9*PI()/180)	=D8+10	=$E$2*SIN(D9*PI()/180)	=C9+E9
10	=A9+10	=B9+10	=$C$2*SIN(B10*PI()/180)	=D9+10	=$E$2*SIN(D10*PI()/180)	=C10+E10

Figure 213: Spreadsheet to determine the resultant waveform when adding two alternating currents

The sine wave can be displayed using the familiar form of cartesian graph, as in figure 214, where the horizontal x-axis represents an angle and the vertical y-axis represents the corresponding voltage. However, the same sine curve can also be displayed in polar form.

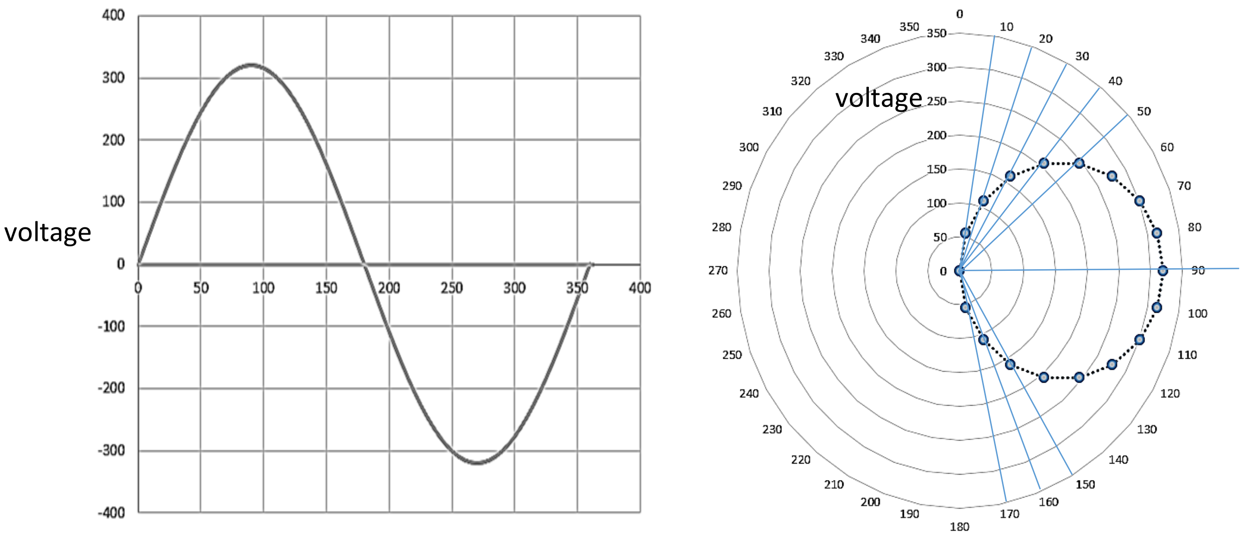


Figure 214: Cartesian and polar representations of a sine curve

If values of the sine function are plotted for each rotation angle from 0 to 180^o on a polar graph, the points are found to lie on a perfect circle. The diameter of the circle represents the amplitude of the wave, and a point with this value occurs at the angle where the wave reaches its maximum peak voltage. In figure 214, this occurs at an angle of 90^o.

Figure 215 shows the effect of adding two sine waves with the same frequency but different phase angles, and amplitudes representing different voltages. The resultant is also a sine wave with the same frequency, but has a different phase and amplitude. In a polar plot, the two sine waves appear as circles with diameters proportional to the two initial voltages, and the resultant appears as a circle with diameter proportional to the resultant.

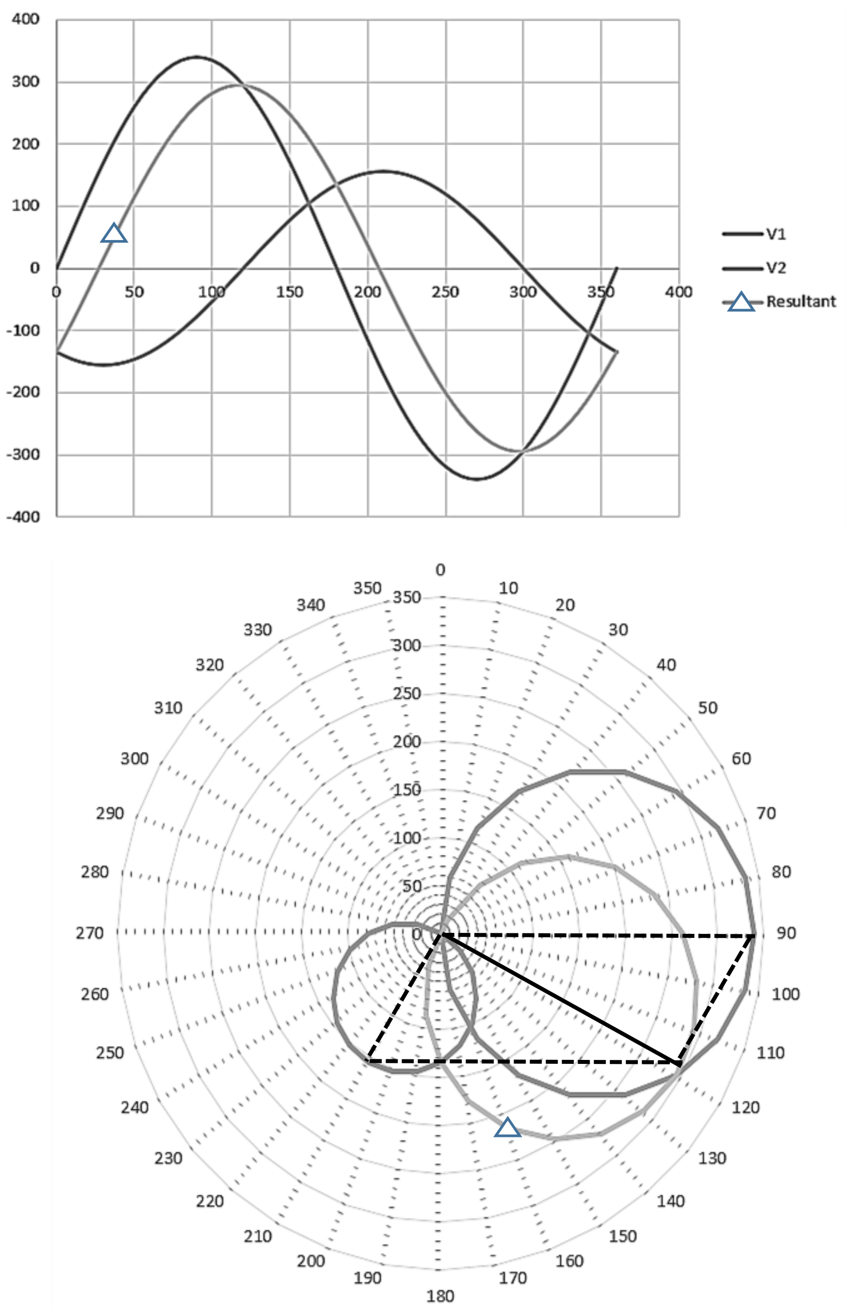


Figure 215: Cartesian and polar graphs showing the addition of two sine functions

It is possible to find the amplitude and phase angle of the resultant by constructing a parallelogram on the polar plot, as shown in figure 215. Sides of the parallelogram represent the amplitudes and phase angles of the original sine waves, with the diagonal of the parallelogram then giving the phase angle and amplitude for the resultant. Indeed, it is possible to use this geometric method directly to determine the resultant of two sine waves, without plotting the actual sine curves.

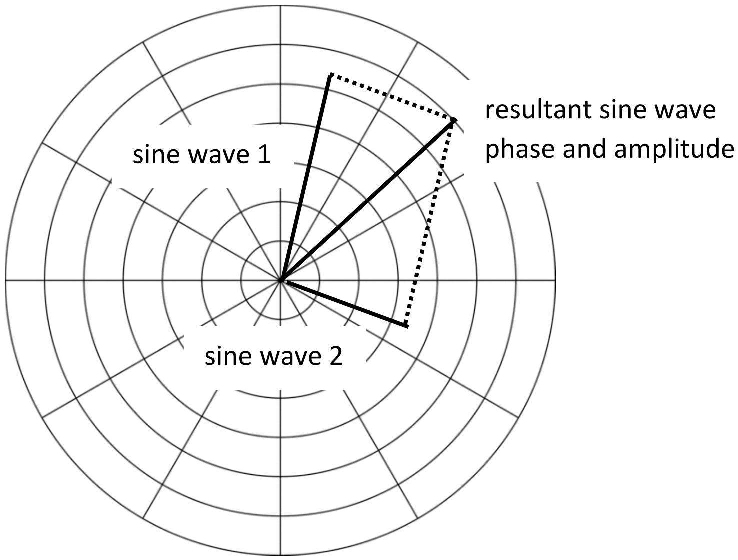


Figure 216: Geometric method for finding the resultant of two sine waves

In the final sections of this chapter, we will examine some specialist types of diagram used in particular vocational areas. We begin with an application from ecology.

Vegetation transects

When investigating plant habitats, it is often useful to produce transect diagrams to illustrate changes of vegetation in response to changing environmental factors. For example, we might wish to investigate the changes in vegetation between the floor of a valley and a hill summit in response to changing soil type, or between the margin of a lake and the drier ground away from the water's edge in response to changes in soil moisture.

In this example, we will look at a vegetation transect across the area of sand dunes at Morfa Harlech, North Wales. Morfa Harlech developed as a spit of shingle and sand carried by longshore drift. An area of salt marsh accumulated behind the spit. At low tide, a large expanse of sand is exposed on a gently sloping beach. This sand can be dried out and blown onto the coast by strong westerly winds, creating the dune system.

A series of parallel ridges of dunes have been produced by seawards migration of the dune system, with progressively older dunes lying inland of the current active dune line.

The vegetation succession along a transect line across the dune system has been investigated by geography students.

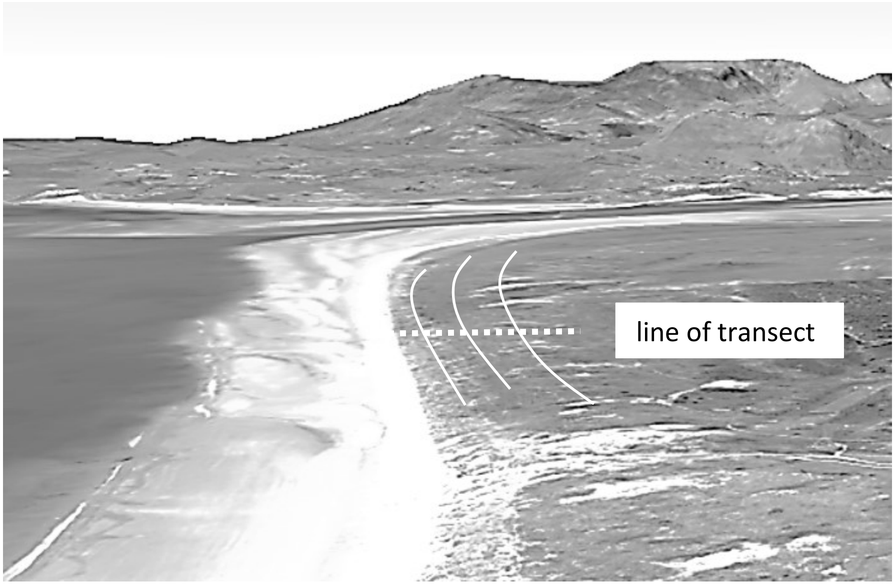


Figure 217: Morfa Harlech sand dunes, showing the transect location

A profile of the ground surface along the transect line can first be surveyed using levelling techniques as described in chapter 7.

At intervals along the transect line, the percentage cover for different plant species is estimated. A convenient way to do this is to use a quadrat frame, divided by strings into a 5-by-5 grid. Each small square then represents 4% of the quadrat area. Plants are identified by means of reference books.

Figure 218: Plant quadrat with string grid

Plant distribution along the transect line can be represented as a series of kite diagrams. A series of key species which illustrate the vegetation succession across the dune system can be selected. Kite diagrams can be conveniently created using a spreadsheet.

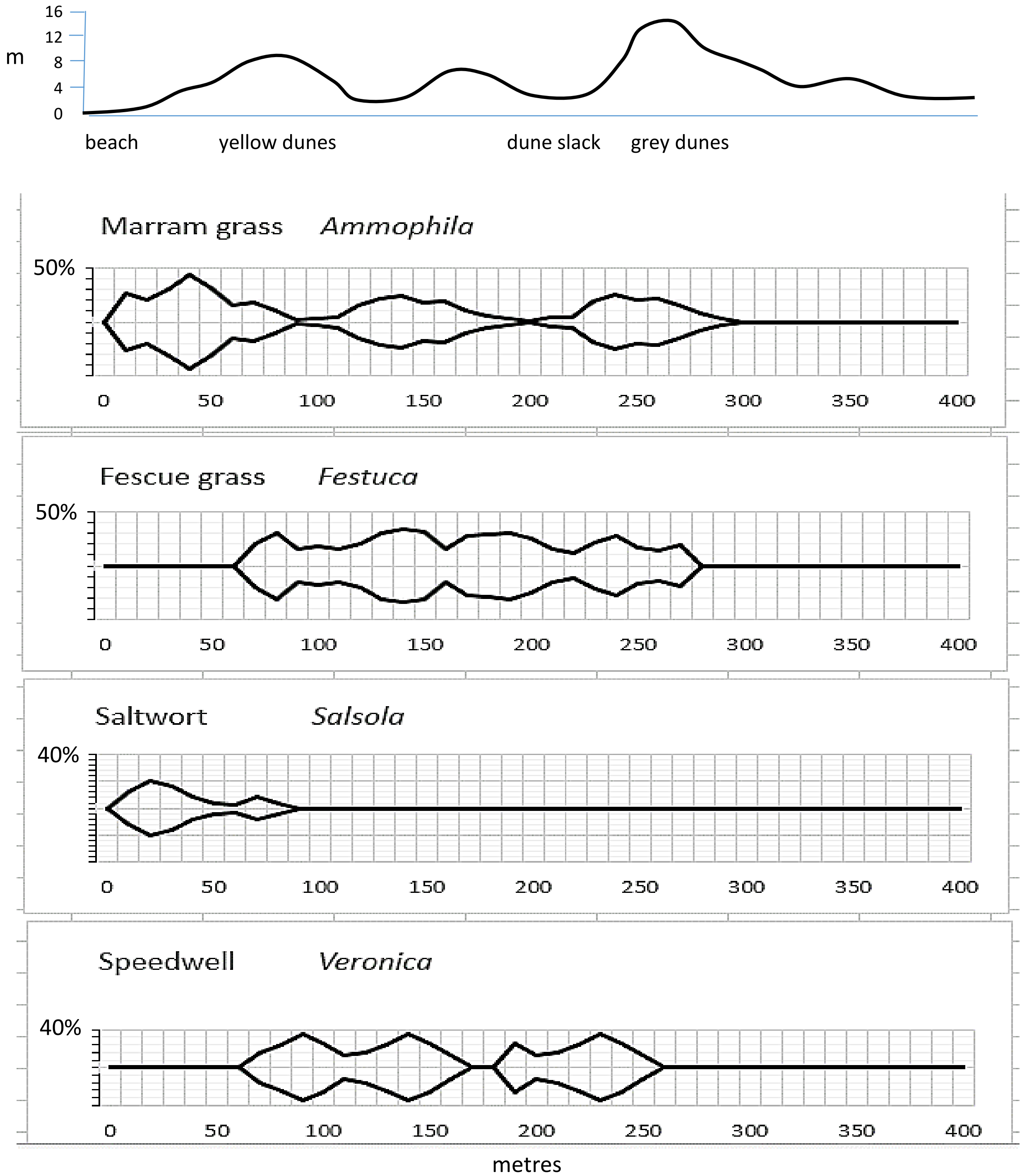


Figure 219: Morfa Harlech sand dune profile and kite diagrams of plant distribution

Actively accumulating yellow dunes, produced by sand blown on shore from the exposed beach at low tide.

Young dunes are stabilised by Marram grass.

Saltwort (*Salsola*)

Ridges of older grey dunes lie behind the active yellow dunes.

Ridges are separated by the damper valleys of the dune slacks.

Speedwell (*Veronica*) and other flowering plants colonise the stable older dunes and dune slacks.

Figure 220: Features of the sand dunes at Morfa Harlech

Sand is supplied to the Morfa Harlech dune system from glacial deposits on the sea bed offshore, and from the Dwyryd estuary to the north.

The loose sand of the first line of dunes is stabilised by early colonising species such as saltwort, sandwort and sea rocket which can tolerate high salinity. Strong root systems of marram grass further stabilise the yellow dunes. As nutrients are added to the sand, the range of plant species can increase. High sand dunes still present a hostile environment for all but the hardiest of plants due to the dry, unstable sandy proto-soil which is poor in nutrients. Between the ridges of the dune system, however, dune slacks occur where growth conditions for plants are more favourable. A greater diversity of flowering plants is seen in these valleys.

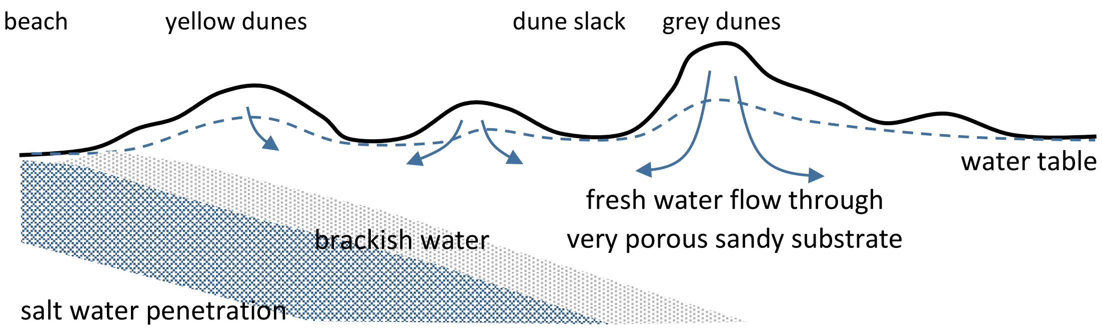


Figure 221: Hydrology of the dune system

Environmental factors along the transect line can be measured. Of particular interest are soil moisture content, soil acidity, relative humidity of the air, and the wind speed which is important in controlling the movement of wind born sand. A scientific model can then be developed to explain the evolution of the dune succession.

In some ecological systems, more complex vegetation relationships need to be displayed. We will examine a method for recording vegetation transects through woodland.

At any particular point within a woodland, a number of layers of vegetation may be present. This will be particularly true of older woodlands in areas of high rainfall where biological productivity is high. We might identify:

- A tree layer, made up from the tallest, most mature trees such as oak or beech. These form the canopy of the woodland.
- A shrub layer. This layer consists of younger individuals of the dominant trees, together with smaller trees and shrubs such as rhododendron and holly which are adapted to grow under lower light conditions.
- A herb layer, such as ferns, grasses and flowering herbs. It is best developed where substantial amounts of light reach the woodland floor, for example in clearings.

- A ground layer, composed of plants which grow in close contact with the soil, such as mosses and fungi.
- Epiphytes, which grow above the ground on tree trunks and branches, such as mosses, lichens and some ferns.

Figure 222: Oak woodland supporting multiple layers of vegetation.

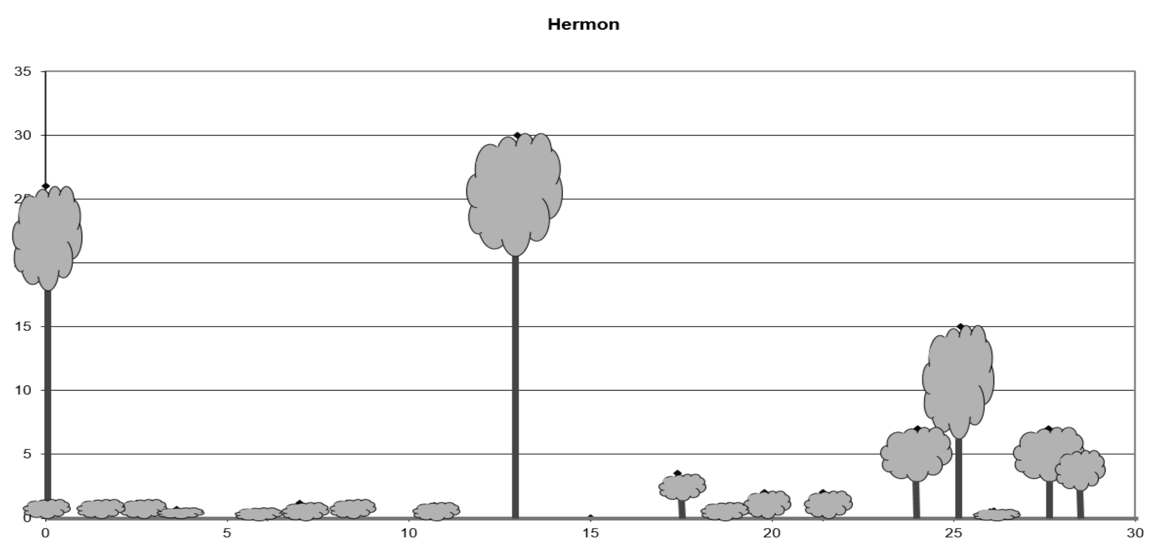
A woodland transect can be developed on a spreadsheet using graphics shape symbols to represent trees and shrubs, as in the examples in figure 223.

Trees, shrubs, herbs and ground vegetation occurring along a transect line are recorded. In the case of trees, the heights to the base and top of the canopy are estimated, if necessary by use of the trigonometrical method described in chapter 6. The canopy diameters of trees and shrubs are also estimated.

The example transects given here use a normal vertical scale, but a logarithmic scale can be used if greater detail of lower vegetation layers is required.

Measured data can give more precise insight into the processes which affect woodland structure, such as:

- Land use change during farming or forestry operations.
- Position of woodland near the base or top of a hill slope, and effects of slope angle.
- Local microclimate, such as the high rainfall over some Meirionnydd oak woodlands which creates temperate rain forest.



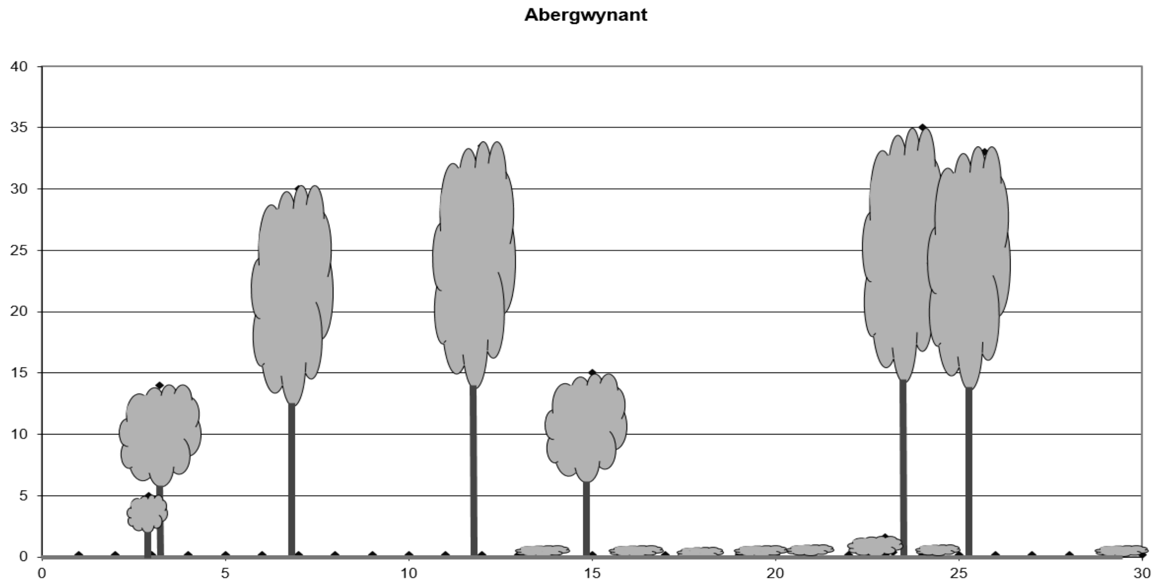


Figure 223: Transects through younger (above) and older (below) oak woodland

Of particular interest is the sequence of vegetation changes which occur as a clear felled area or grassland is left to revert naturally to woodland:

Grasses and tall herbs become established, and provide shelter for seedlings of fast growing tree species such as birch.

As the forest develops, the early colonising trees in turn provide shelter for the slower growing and longer lived climax species such as oak and beech. The extent of the canopy cover will determine how much light can penetrate to the forest floor, and the amount of plant growth in the lower layers. Oak produces a more open canopy than beech, so oak woodlands are particularly rich in herb and ground vegetation.

Mature trees may be blown down during storms or simply die naturally, creating gaps in the canopy which allow light to penetrate and support growth of the lower layers of vegetation in small clearings.

As mature trees develop, a rich collection of mosses, lichens and ferns can grow on their trunks and branches, particularly where sufficient light and a high humidity are present. Fungi have a role in breaking down fallen trees and returning nutrients to the soil.

As the woodland succession progresses, there is an increase in biodiversity of both the plants and the animal species which are dependent on woodland habitats. Soil organic matter and nutrient levels increase during the sequence, which in turn leads to greater biological productivity. The multiple layering of the vegetation allows overall biomass to increase as the woodland develops.

For our next examples of specialist diagrams, we will examine the use of diagrams to record sequences of rock strata.

Graphic log sections in geology

A graphic log provides a method of representing thickness of beds of rock, their grain size and sedimentary features. This can provide valuable evidence of the geological environment in which a sequence of rocks were formed.

Figure 224: Recording a measured log section through a sequence of Cambrian grits, Barmouth

To produce a graphic log section, an outcrop of rock is chosen where there is easy and safe access to examine the sequence of strata. A measuring tape is positioned so that the thickness of each rock layer between the bedding planes can be recorded. In addition to recording thickness, the maximum grain size of the material is measured at the top and bottom of each bed. Grain size is also recorded at intervals within a thick bed of rock, or where obvious changes in grain size occur.

Grain size is measured using the phi scale, as discussed earlier in chapter 6. Example phi values are:

phi scale values

4	3	2	1	0	-1	-2	-3	-4	-6	-8
MUD				SAND				GRAVEL		
clay	silt	v fine	fine	medium	coarse	v coarse	granules	pebbles	cobbles	boulders

Data can then be transferred manually to graph paper to produce the log section, or a computer graphics application can be used, as in figure 226. Each layer of rock is plotted with the appropriate thickness on the vertical scale, and a horizontal width representing the grain size. This method of illustration reflects the actual appearance of the rock sequence in a weathered outcrop. Coarser sand and pebble beds are generally more resistant to erosion than finer sand, silt and mud bands, so extend further out from the rock face. Shading can be added to represent pebbles or layering present in the beds.

The sequence shown in figure 226 is a sequence of deep water grits and sandstones of Lower Cambrian age, belonging to the Barmouth Grit formation. It is thought that these sediments were laid down from clouds of turbulent sediment which discharged down a continental slope onto the deep ocean floor. Similar deposits, known as ***turbidites***, are known to form at the present day on the ocean floor off shore from the mouths of major rivers. Examples of turbidite deposits are found in the north Atlantic off the mouth of the St Lawrence River, and in the Gulf of Mexico off the mouth of the Mississippi.

Studies have shown that each cloud of turbidite sediment which is discharged onto the sea bed can lay down a particular sequence of beds. These are designated with reference letters from A to E:

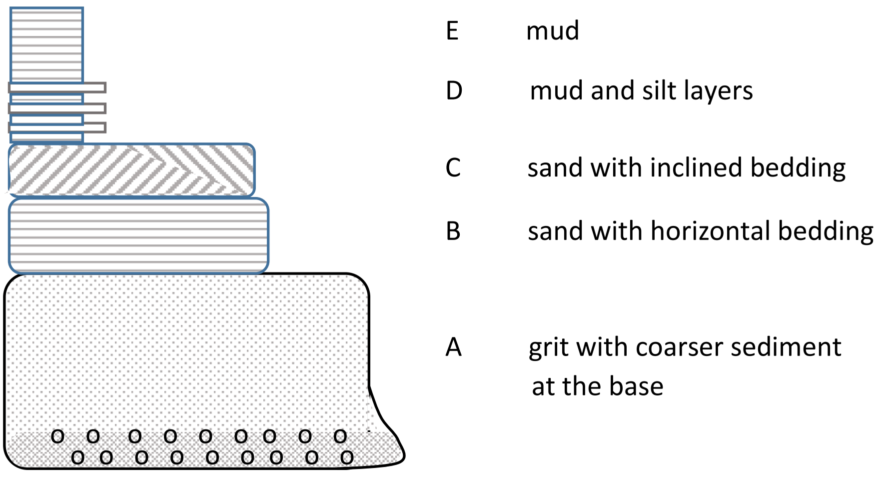


Figure 225: Sequence of sediments which can be laid down by a turbidite flow

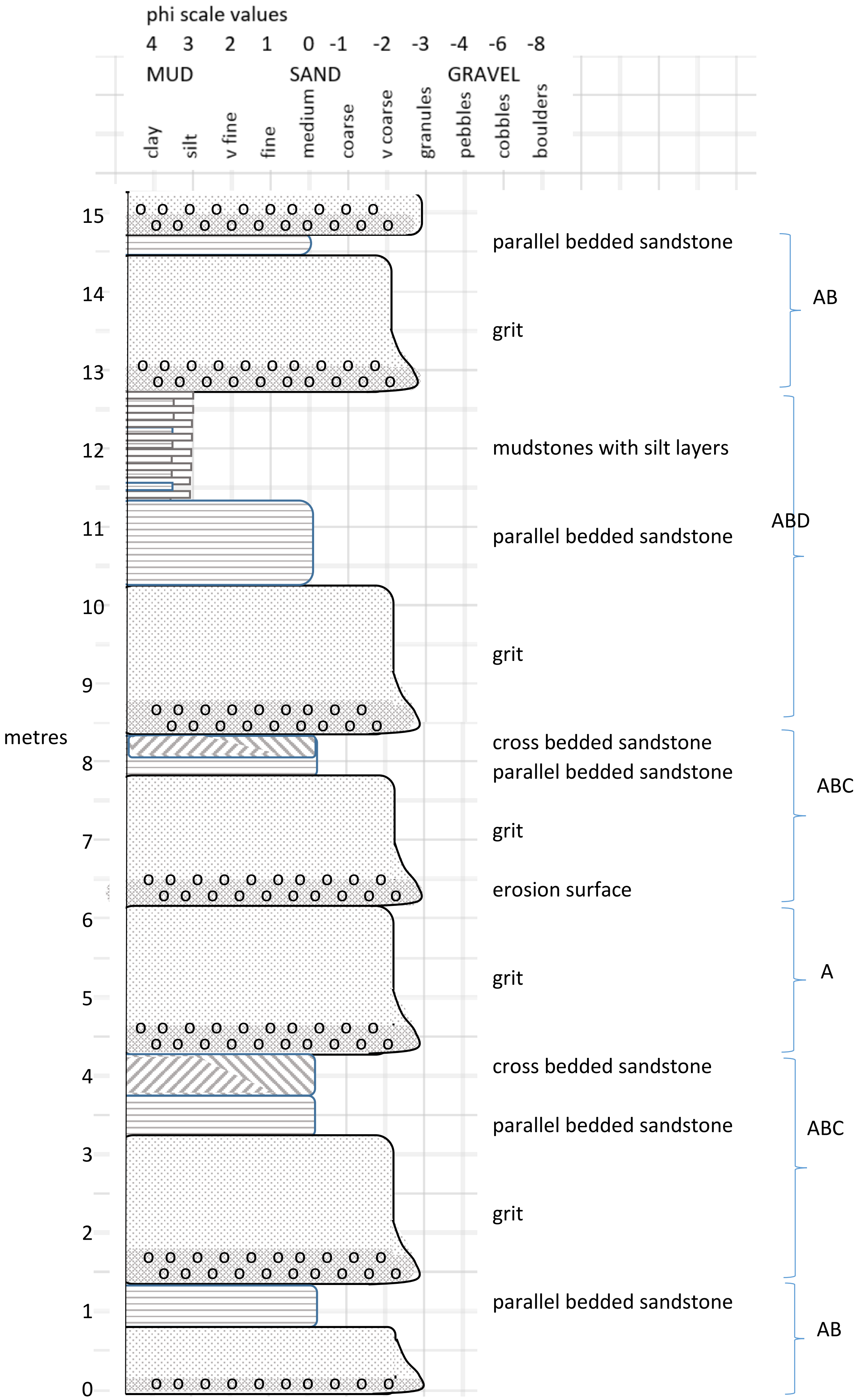


Figure 226: Graphic log section in Lower Cambrian grits, Barmouth

Not all of the layers of the full turbidite sequence may be present in any particular instance.

> This may be due to another cloud of sediment rapidly following the initial sediment discharge onto the sea floor, so that there is no time for the finer sands and muds from the first flow to settle. This may result in a repeated A unit in the sequence.
>
> A cloud of coarse sediment discharged onto the sea floor may have an erosional effect, removing any finer silts and muds which were laid down after the previous flow. In this case, the D and E units may be missing from the final sequence.

Evidence of both of these mechanisms can be identified in the graphic log section above.

The use of graphic log sections is not restricted to recording sediments laid down by river or sea processes, but can also be used to interpret volcanic deposits. Figure 228 shows a measured section through ashes erupted from the ancient Cader Idris volcanic centre during the Ordovician period. Individual beds vary in texture from fine dust to very coarse deposits containing large blocks of rock which exploded from the volcanic vent (figure 227).

Figure 227:

Volcanic ashes of Ordovician age, Cader Idris

Geological evidence suggests that during the Ordovician period, Wales was an area of shallow sea with volcanic islands intermittently erupting.

> In the dormant periods between volcanic events, rivers would carry finer volcanic sediment into the sea where it could be deposited as layers of sand and mud.
>
> At the start of each volcanic episode, explosive eruptions would throw rock debris and lava bombs high into the air, along with clouds of finer ash. The finer material might settle out from the atmosphere over a period of days or weeks following the eruption.

Evidence of these processes can be identified in the graphic log section of rocks from the Llyn Cregennen area of Cader Idris shown in figure 228.

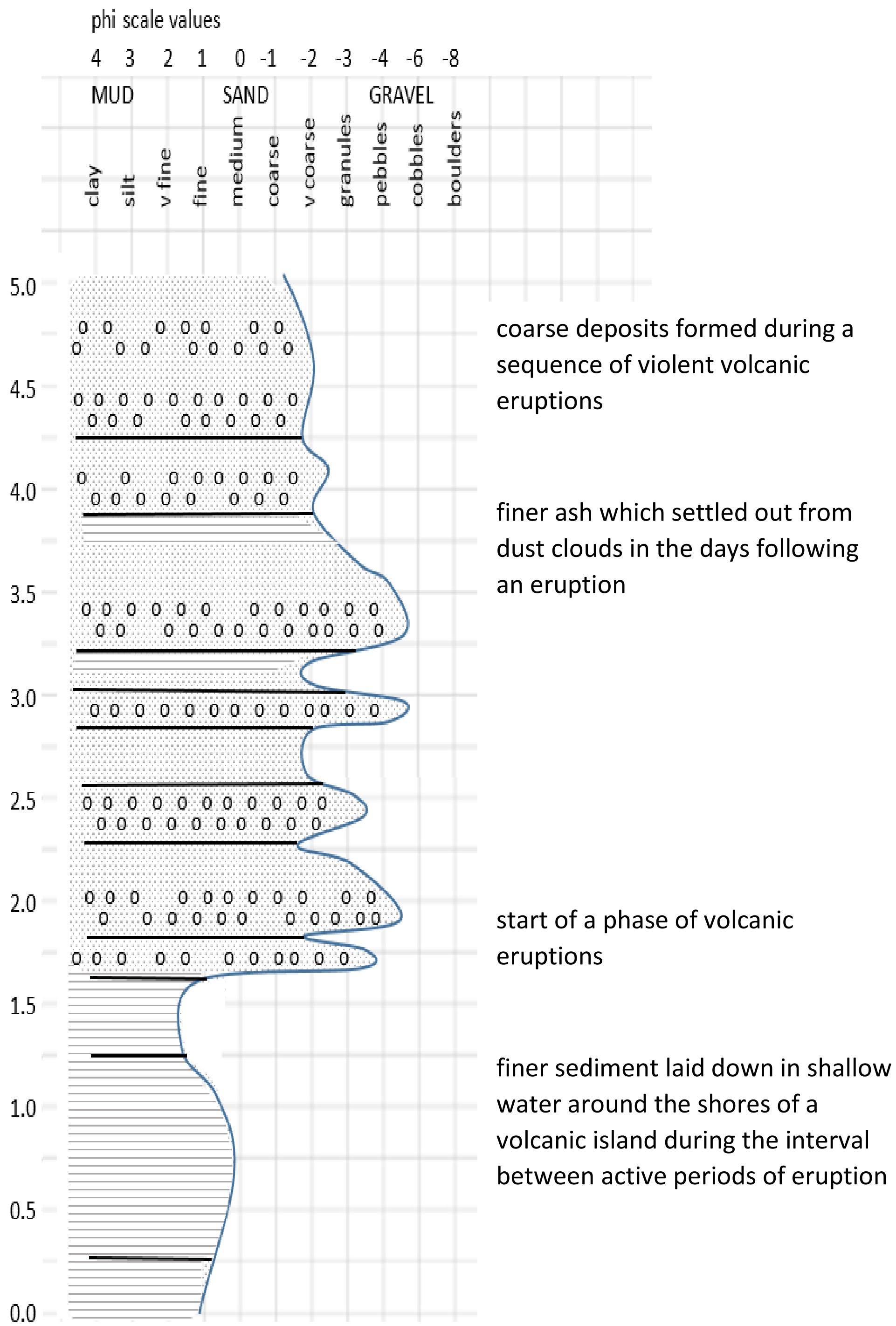


Figure 228: Graphic log section in the Cefn Hir formation at Llyn Cregennen

In the next sections, we will look at ways in which measured data can be added to maps as a way of identifying and interpreting patterns. Analysis of patterns can be an important component of problem solving.

Traffic flow

A convenient way to illustrate traffic flows is by lines of proportional thickness on a map. Lines may represent numbers of people or numbers of vehicles travelling between locations. It is sometimes appropriate to calculate a ***passenger car equivalent*** value when carrying out a traffic survey, so that different weightings are allocated to cars, bicycles, taxis, buses and other vehicle types according to the average number of persons transported. Maps may be created for particular periods of the day, for example: to compare rush hour traffic with quieter periods. We may look at flows in a particular direction, for example: to evaluate the flows into a town centre in the morning, and out of the town centre at the end of the working day.

Maps may be created to answer particular research questions, such as determining the number of pedestrians or cyclists following a particular route, or showing the numbers of public transport services available. We will look at two examples here.

Our first map illustrates the numbers of train services operating over different routes in Wales and the borders during a typical weekday. The numbers of services in one direction are found to be:

From	To	Number of services
Birmingham, Shrewsbury	Chester	32
Cardiff, Hereford	Shrewsbury	30
Carmarthen, Swansea	Cardiff	52
Holyhead, Llandudno	Chester	30
Llandudno	Blaenau Ffestiniog	5
Shrewsbury	Aberystwyth	16
Machynlleth	Pwllheli	8
Shrewsbury, Llandrindod	Swansea	5
Rhymney Valley	Cardiff	50
Merthyr	Cardiff	45
Fishguard, Milford Haven	Carmarthen	12

The flow map was created by first copying the outline of the coast and the railway routes using graphics shape components in Microsoft Word. The thicknesses of the curves representing sections of the rail route ware then adjusted using the proportion:

50 journeys = 20 point line thickness

Analysis of the map gives a clear overview of the rail transport provision. A majority of services are concentrated along the South Wales and North Wales coasts and the Welsh borders. Relatively few rail services are provided to serve the sparsely populated central region of the country.

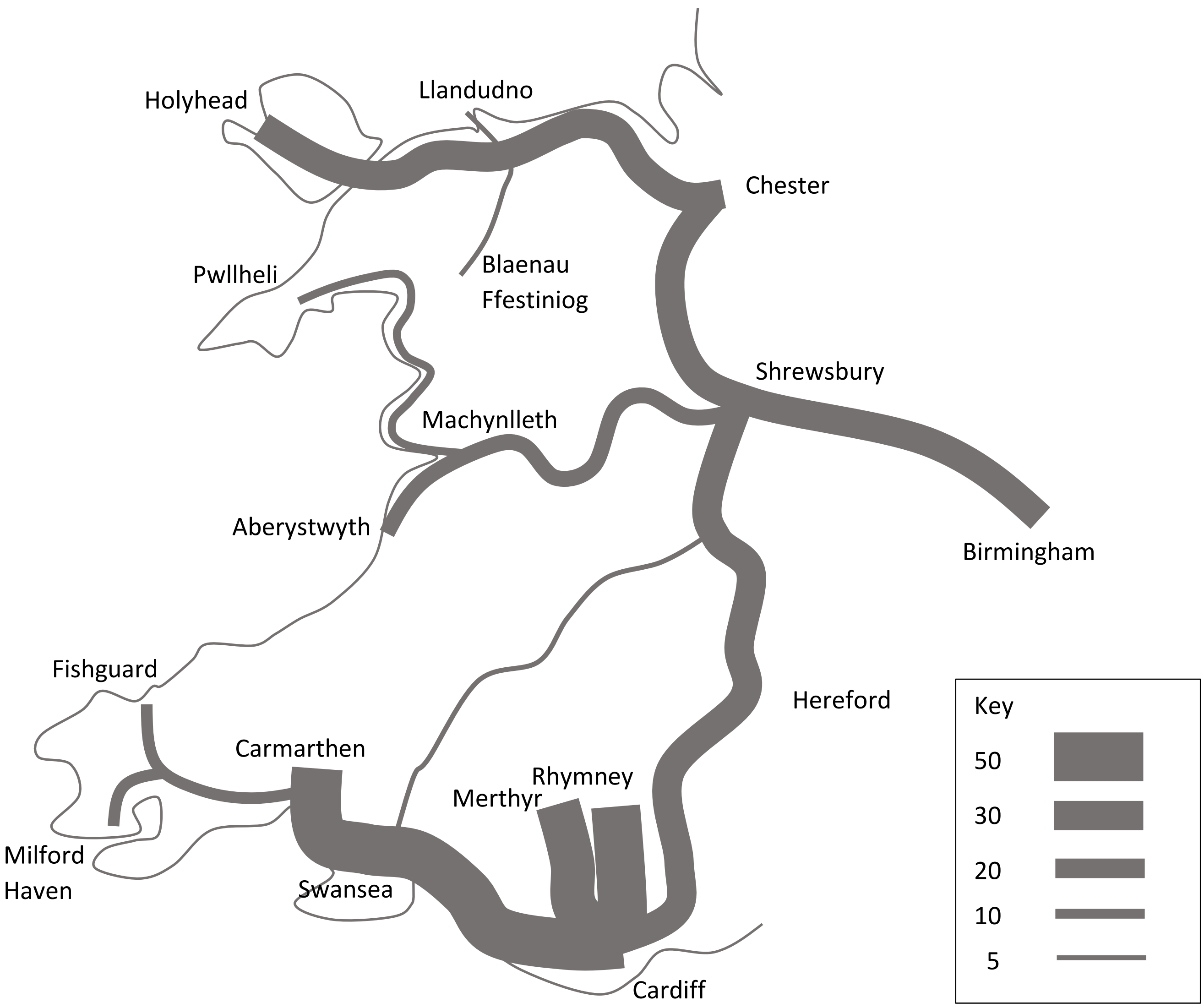


Figure 229: Number of railway services in one direction along routes in Wales on a weekday

A second flow map example illustrates numbers of walkers and cyclists using the Mawddach Trail. This is an 8 mile path following the line of a disused railway line from Dolgellau to the coast, then crossing the Mawddach Estuary to Barmouth along a walkway over the Barmouth railway viaduct (figure 230).

In 2016, at a time of cut backs in public spending, Gwynedd County Council proposed to close the public walkway across the Barmouth viaduct in order to save the costs of maintenance. This proposal was opposed by local businesses in the tourism sector, as it would close the through route for walkers and cyclists along the Mawddach trail and have an adverse effect on visitor numbers. To evaluate the amount of usage of the Mawddach trail, students carried out a survey of combined numbers of walkers and cyclists using different sections of the route on a Saturday in April. Counts were made for each of the directions of travel. Results are displayed in figure 231 below.

Figure 230: Barmouth railway viaduct at the mouth of the Mawddach estuary

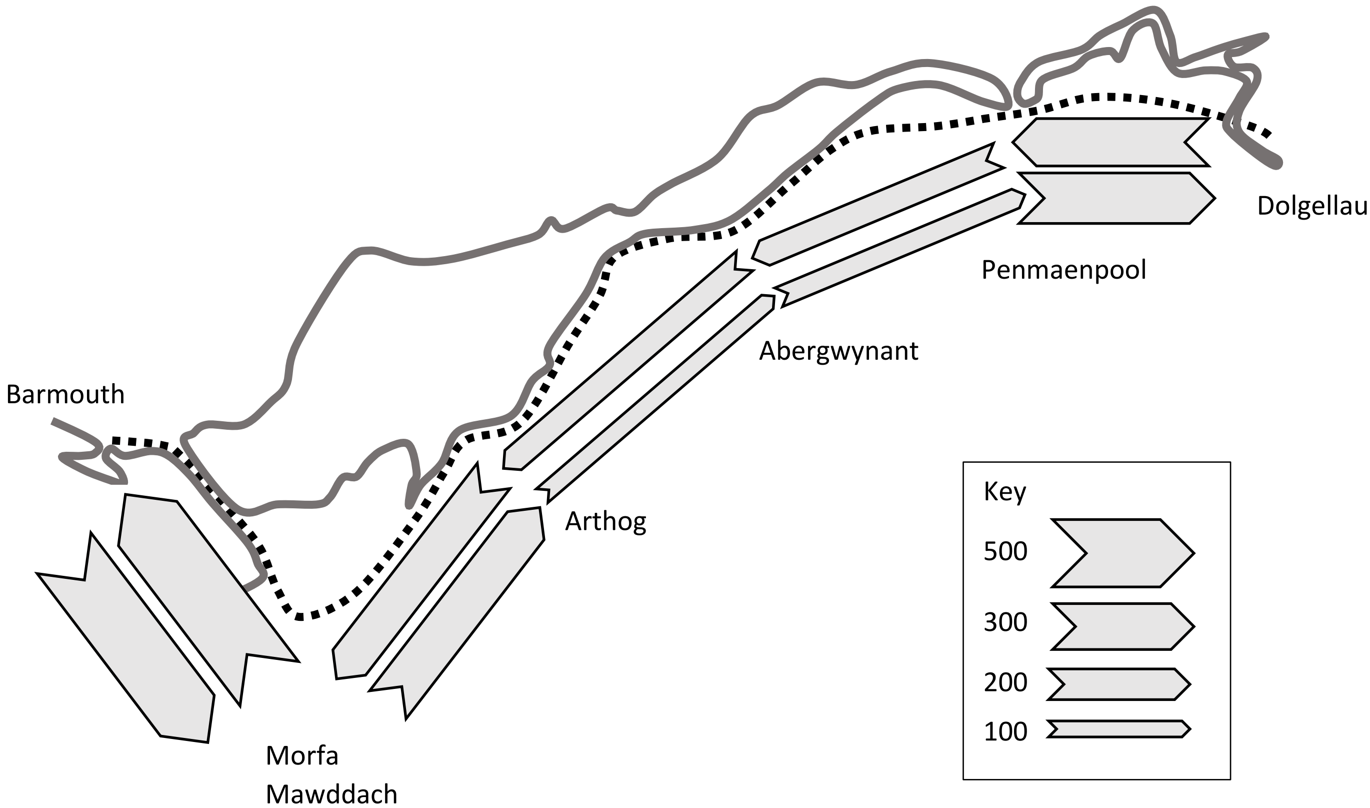


Figure 231: Survey of usage of the Mawddach Trail by walkers and cyclists on a Saturday in April

Results of the survey indicate very heavy usage of the Barmouth viaduct by walkers and cyclists, though it appears that only a small proportion travel along the complete length of the Mawddach trail between Barmouth and Dolgellau.

Urban land use

Students of geography or travel and tourism may carry out projects to examine the structures of towns, and make proposals for developments to benefit the people who live and work in the towns or visit for shopping or leisure.

A first stage in an urban study is often to produce a land use map to identify functional zones of the town. Several theoretical models have been proposed for the structures of towns and cities. Perhaps the simplest is the **concentric zone model** proposed by **Burgess**.

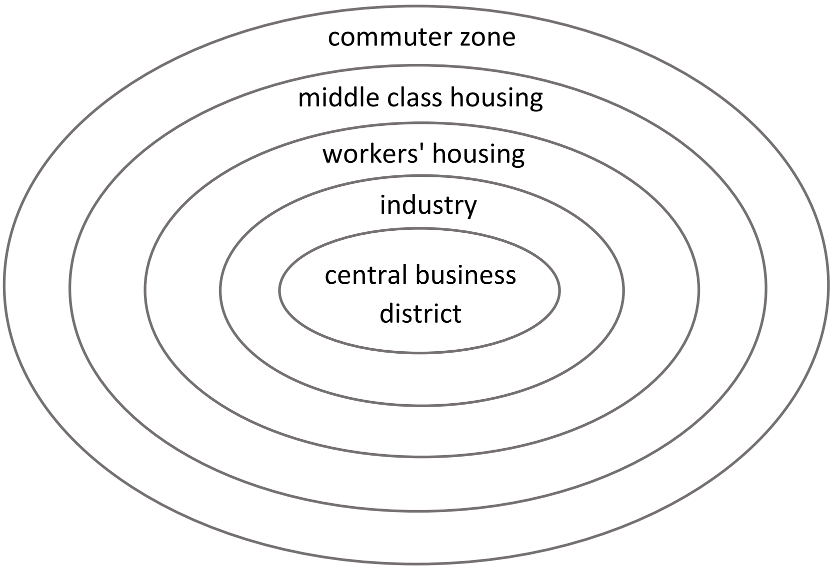


Figure 232: Concentric zone land use model of Burgess

In this model, the core of the town is occupied by the central business district. This is the most favourable area for retail and commerce, but the cost of land and buildings is greatest. This is the area where nationally important businesses will site their branches, including: banks, large stores, cinemas and other places of entertainment. Local and national government organisations often have offices in the central business district.

The zone bordering the central business district is traditionally one of factories and industry. Around this would be workers' housing, often in terraced streets which developed during the period of growth accompanying the industrial revolution.

Beyond the workers' housing would be the larger houses of middle class professionals. As cities expanded and transport links improved, the surrounding towns and villages could provide a commuter zone with a lower cost of housing than the city.

Changes in the ways that people use cities, and a greater appreciation of the effects of geographical features such as rivers on urban development, led to the more complex **sector model** of **Hoyt**.

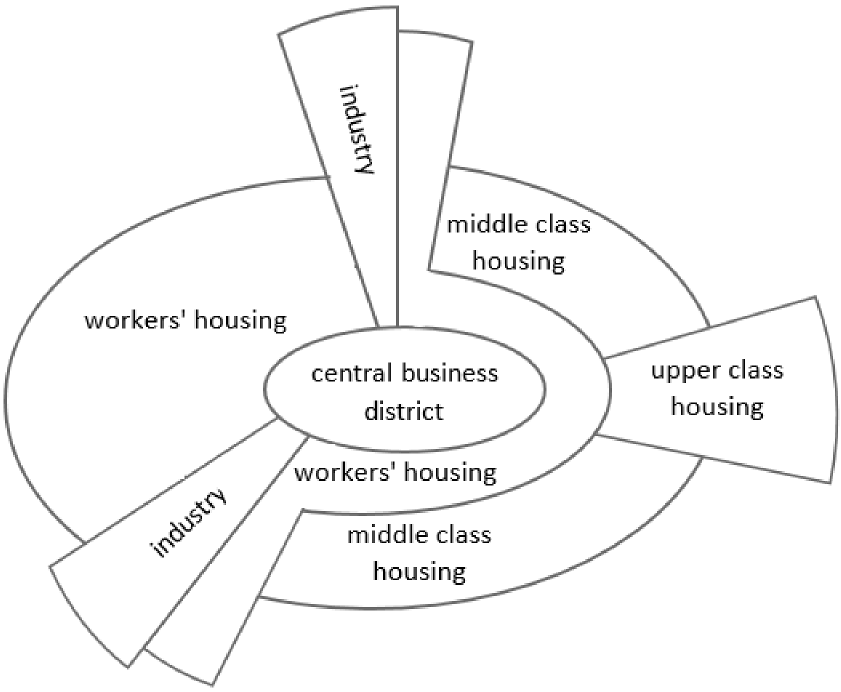


Figure 233: Sector land use model of Hoyt

The Hoyt model has some similarities with the Burgess model, with the core of the city again occupied by the central business district. The main difference is that zones of industry and the accompanying houses for workers now follow corridors from the centre of the city to the surrounding region. These corridors might, for example, be a canal or railway with good facilities the transport of goods and raw materials. In a similar way, higher quality housing might follow particular geographical features, such as a picturesque river valley or land with good access to an important road or rail link.

Students, as well as considering the extent to which a particular town follows the theoretical models, might make an assessment of the current state of the town, in terms of:

- Quality of the shopping centre, considering the number of large and small shops, the range of goods and services provided, and the proportion of vacant premises.
- The standard of housing, in terms of the range of different housing types, the state of repair of the buildings, the state of upkeep of neighbourhoods and freedom from crime and antisocial behaviour.
- The provision of public transport, and conditions for traffic flow and parking.
- Public services, in terms of provision for education, health care and other essential functions.
- Recreation, including the provision of open spaces and the provision of entertainment venues such as cinemas and theatres.
- The extent to which expansion or redevelopment is taking place, considering for example the building of new shopping facilities in the town centre or outside the town.

The starting point for the town study is to prepare a land use map. Suitable base maps showing streets and buildings are readily available from Internet sites such as Google Maps and Bing Maps.

A standard colour coding scheme which is often used for urban land use studies is:

Residential	Brown
Industrial	Grey
Commercial (including retail)	Blue
Entertainment	Red
Public buildings	Yellow
Open space	Green
Transport	Black
Services	Orange

Carrying out a land use study of Dolgellau in North Wales (figure 234) can provide a better understanding of the structure and development of the town:

A. A small central business district contains banks, chemists, hotels and a range of small shops selling food, clothing and household goods, in addition to shops catering mainly for tourist visitors.
B. The older part of the town, now a historic conservation area, consists mainly of small terraced cottages built originally for workers of the woollen mills of the town during the 18th and 19th centuries.
C. A number of businesses requiring larger amounts of land are located on the edge of the town. In the photograph are seen: (right) a petrol station and garage, (centre left) farmers' animal market, and (left) a supermarket.
D. Larger Victorian villas occupy an attractive wooded hillside above the town.

A small modern industrial estate occupies land in the river valley conveniently near to the centre of the town, but historically considered unsuitable for development due to a high risk of flooding. This risk has been reduced in recent years by the construction of a flood defence scheme.

The town has expanded within the past century, with the construction of additional housing around the margins of the old town.

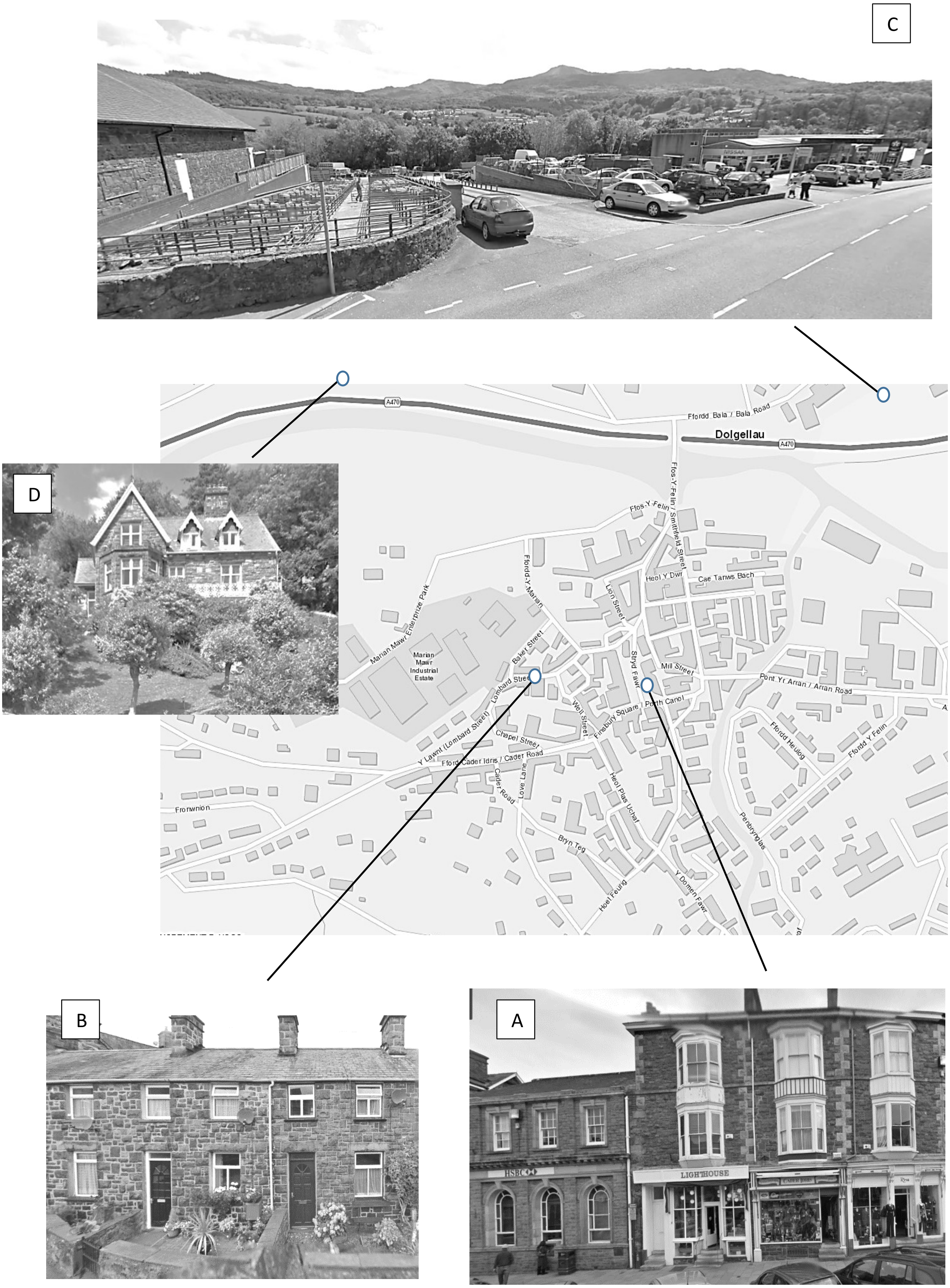
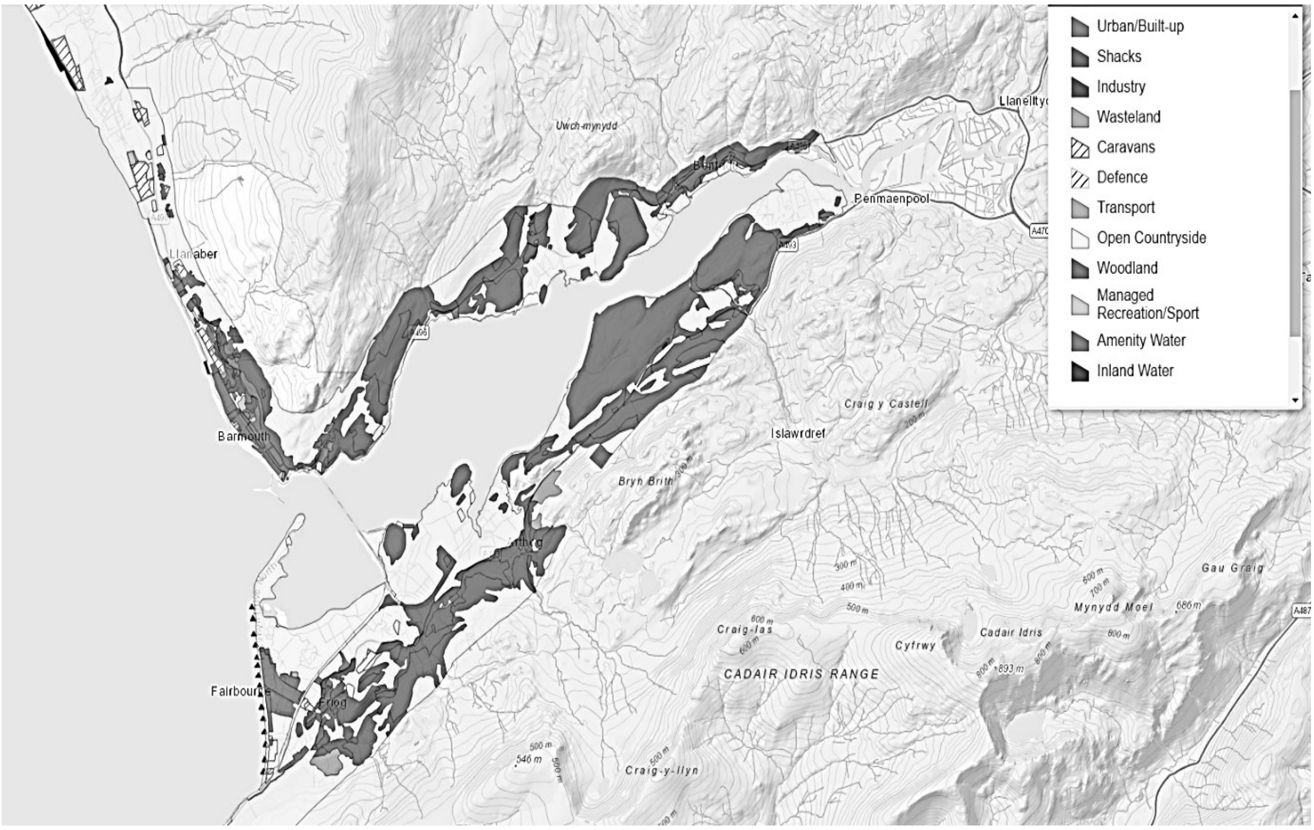


Figure 234: Observations to produce a land use survey of the town of Dolgellau

'Mapping our Shores' is a major land use mapping project undertaken by the National Trust, with results published on the Internet. A series of categories of land use have been identified, as in the example below for the Barmouth and Fairbourne areas.

Caravans

Urban/
Built-up



Managed
Recreation
/Sport

Open
Countryside

Figure 235: Example land use map from the ***Mapping our Shores*** project, National Trust